Physics Cup 2022
Problem 1

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At some point in the experiment, let $B_\infty \overrightarrow{u_z}$ be the applied magnetic field, measured far from the ball. We will determine a relation between B_{in} and H_{in} inside the ball by solving the magnetostatics problem: this relation, along with the temperature-dependent hysteresis loop provided, will allow us to fully determine the values of B_{in} and H_{in} for some given temperature, we will finally deduce the magnetisation M from those values.

Due to hysteresis there may be several (B_{in}, H_{in}) solutions possible but assuming that one branch of the hysteresis loop has been chosen, then the magnetostatics problem has only one solution for the space dependency of the fields. It thus suffices that we take notice of a particular solution to ensure we have solved the problem, leaving for later the issue of knowing B_{in} as a function of H_{in}.

For this solution it is reasonable to assume that the fields B_{in}, H_{in} and M be uniform inside the ball, then by symmetry they will be along $\overrightarrow{u_z}$. Furthermore far from the ball we know that the field will look like $\overrightarrow{B}_{\text{dipole}} + B_\infty \overrightarrow{u_z}$, where $\overrightarrow{B}_{\text{dipole}} = \frac{\mu_0}{4\pi r^3}\left(3\left(\overrightarrow{m} \cdot \overrightarrow{u_r}\right)\overrightarrow{u_r} - \overrightarrow{m}\right)$ is the magnetic field created by a dipole with moment $\overrightarrow{m} = \frac{4}{3}\pi R^3 M \overrightarrow{u_z}$ ($\overrightarrow{u_r}$ is a radial unitary vector). We will show that this is actually the exact solution everywhere outside the ball, provided B_{in} and H_{in} verify a certain relation.

We know that the fields we have written are valid solutions to Maxwell's equations both inside and outside the ball: it only remains to be checked that the boundary conditions at the limit of the ball are met. Those can be written $\overrightarrow{H}_{\text{out}} - \overrightarrow{H}_{\text{in}} = -\left(\left(\overrightarrow{M}_{\text{out}} - \overrightarrow{M}_{\text{in}}\right) \cdot \overrightarrow{u_r}\right)\overrightarrow{u_r}$ (here, $\overrightarrow{M}_{\text{out}} = \overrightarrow{0}$). Rearranging and using the general relation $\overrightarrow{B} = \mu_0\left(\overrightarrow{H} + \overrightarrow{M}\right)$ inside and outside the ball yields: $B_{\text{in}} = -2\mu_0 H_{\text{in}} + 3B_\infty$. This is the relation that we wanted and its intersection with the hysteresis curve will provide knowledge of the possible values of (B_{in}, H_{in}).

Hence, when B_∞ is initially raised to ~ 3 T, which is a very large value, the system will be taken out of its linear domain and will saturate. So, when B_∞ is removed, the working point of the system will be as indicated on the figure below, on the upper hysteresis branch (that which is given as a plot). A change in temperature leaves unchanged the relation $B_{\text{in}} = -2\mu_0 H_{\text{in}} + 3B_\infty$ and will only affect which hysteresis curve we should look at on the plot. Therefore, we have: $B_1 = 0.95$ T, $\mu_0 H_1 = -0.48$ T and $B_2 = 0.48$ T, $\mu_0 H_2 = -0.24$ T. From $B = \mu_0(H + M)$ we can compute $M_1 = 1.1 \times 10^6$ A m^{-1} and $M_2 = 5.7 \times 10^5$ A m^{-1}. Since $m = \frac{4}{3}\pi R^3 M$, the change in magnetic moment of the ball due to heating is: $\boxed{\Delta m = -2.4 \text{ A m}^2}$.

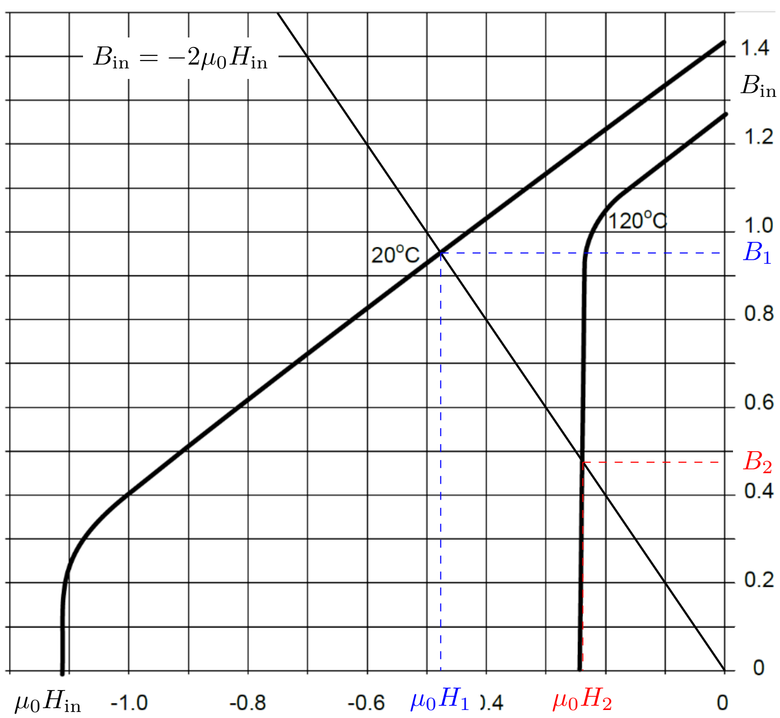

Figure 1: Hysteresis curve and working points