Physics Cup 2022 - Problem 1

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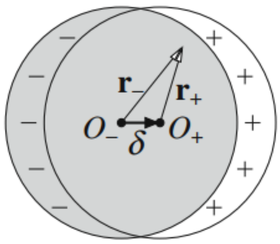
Lemma 1. A dielectric sphere with an uniform polarization $\vec{P}$, the electric field inside the sphere is uniform and equals $\vec{E}_{in} = -\frac{\vec{P}}{3\epsilon_\circ}$.

Proof:
This model is equivalent to the superposition of two "rigid" spheres: one of uniform positive charge density ρ_0, comprising the nuclei of the atoms, and a second sphere of the same radius, but of negative uniform charge density -ρ_0, comprising the electrons.

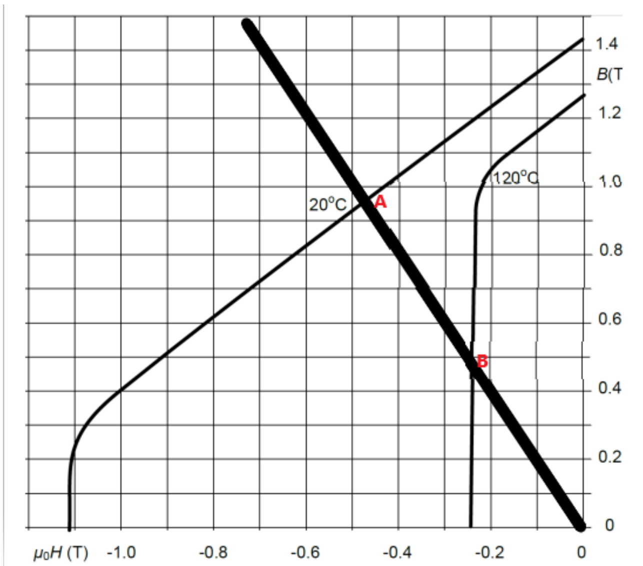


Indeed, the electrostatic field at any point in space is the sum of the fields generated by each charged sphere . The field generated by a single uniformly charged sphere at its interior is $\vec{E} = \frac{\rho_\circ \vec{r}}{3\epsilon_\circ}$, where $\vec{r}$ is the position vector relative to the center of the sphere. Thus, the two spheres generate at their interiors the fields $\vec{E}_\pm = \pm\frac{\rho_\circ \vec{r}_\pm}{3\epsilon_\circ}$, respectively, $\vec{r}_\pm$ being the position vectors relative to the two centers. We assume that the centers are located on the x axis at points $O_+ \equiv (+\frac{\vec{\delta}}{2}, 0, 0)$ and $O_- \equiv (-\frac{\vec{\delta}}{2}, 0, 0)$. We thus have $\vec{r}_\pm = \vec{r} \mp \frac{\vec{\delta}}{2}$, where r is the position vector relative to the origin O $\equiv$ (0,0,0). The total field in the overlap region is:
$\vec{E}_{in} = \frac{\rho_\circ}{3\epsilon_0}(\vec{r} - \frac{\vec{\delta}}{2}) - \frac{\rho_\circ}{3\epsilon_0}(\vec{r} + \frac{\vec{\delta}}{2}) = -\frac{\rho_\circ \vec{\delta}}{3\epsilon_0}$
The internal field E_{in} is thus uniform and proportional to $-\vec{\delta}$.
The polarization vector of the sphere is:
$\vec{P} = \rho_\circ \vec{\delta} = -3\epsilon_\circ \vec{E}_{in} \Rightarrow \vec{E}_{in} = -\frac{\vec{P}}{3\epsilon_\circ}$
Lemma 2. A sphere of radius R has a uniform and permanent magnetization $\vec{M}$, the magnetic field inside the sphere is uniform and equals $\vec{B} = -2\mu_\circ \vec{M}$.

Proof:
- Maxwell 's equations in vacuum:
+ The electrostatic equations $\nabla \cdot \vec{D} = 0$; $\nabla \times \vec{E} = 0$
+ The magnetostatics equations $\nabla \cdot \vec{B} = 0$; $\nabla \times \vec{H} = 0$
- Relations:
+ $\vec{D} = \epsilon_\circ \vec{E} + \vec{P}$
+ $\vec{B} = \mu_\circ(\vec{H} + \vec{M})$ (1)

- The spatial distribution of $\vec{M}$ is the same as that of $\vec{P}$ in **Lemma 1**, and the boundary conditions for $\vec{H}$ are the same as for $\vec{E}$.
From all the above analogies $\Rightarrow \frac{\epsilon_\circ \vec{E}}{\vec{P}} = \frac{\vec{H}}{\vec{M}} = -\frac{1}{3} \Rightarrow \vec{M} = -3\vec{H}$ (2)
From (1) and (2) $\Rightarrow \vec{B} = -2\mu_\circ \vec{H}$

Apply to solve the problem.
At any temperature T, the operating point of spherical ball made from a NdFeB alloy is the intersection of the left upper quadrant $(H < 0, B > 0)$ of the hysteresis loop and the linear line $B = -2\mu_\circ H$.



From the graph we see that:
The operating point of spherical ball $(\mu_\circ H, B)$ is (-0.475, 0.95) at $20°C$ and (-0.25, 0.5) at $120°C$
The magnetic dipole moment of the ball: $m = M \times \frac{4}{3}\pi R^3$
From (2) $\Rightarrow m = -4\pi R^3 H$
So when the temperature increases from $20°$ to $120°$, the magnetic dipole moment of the ball changes: $\Delta m = -4\pi R^3 \Delta H = -2.25(A/m^2)$