Firstly, the ferromagnet will be nearly maximally magnetized in the positive **B** direction when the outer applied field is 3T. Afterwards, the state of the ferromagnet will lie on the upper, $20°C$ curve.

Now, let us study the state of the spherical magnet under no external field. Inside the material, we have

$$\mu_0 \mathbf{H_{in}} = \mathbf{B_{in}} - \mu_0 \mathbf{M} \Longrightarrow \mathbf{B_{in}} = \mu_0(\mathbf{H_{in}} + \mathbf{M}), \tag{1}$$

while outside we know that

$$\mathbf{B_{out}} = \mu_0 \mathbf{H_{out}}. \tag{2}$$

We know that the boundary conditions state that

$$\begin{aligned} \mathbf{B}_{\mathbf{in},\perp} &= \mathbf{B}_{\mathbf{out},\perp}; \\ \mathbf{H}_{\mathbf{in},\parallel} &= \mathbf{H}_{\mathbf{out},\parallel}. \end{aligned} \tag{3}$$

Let us consider a patch of the sphere at position angle θ with respect to the point which is furthest on the sphere along the direction of **B**. Inside the magnet, both fields are along the same direction, namely along the axis of magnetization of the sphere, which is at an angle θ with respect to the local normal at the patch we are considering. Therefore, we will have

$$B_{in,\perp} = B_{in} \cos\theta, \tag{4}$$

so that

$$B_{out,\perp} = B_{in} \cos\theta. \tag{5}$$

Similarly, we will have

$$H_{in,\parallel} = H_{in} \sin\theta, \tag{6}$$

so that

$$H_{out,\parallel} = H_{in} \sin\theta. \tag{7}$$

However, we know the following:

- Inside the sphere, B takes a specific value as a function of $\mu_0 H$; let us call it $B(\mu_0 H_{in})$:

$$B_{in} = B(\mu_0 H_{in}) \Longrightarrow \frac{B_{in,\perp}}{\cos\theta} = B(\mu_0 H_{in}) \Longrightarrow B_{out,\perp} = B(\mu_0 H_{in}) \cos\theta; \tag{8}$$

- while outside of it,

$$\begin{aligned} B_{out,\parallel} &= \mu_0 H_{out,\parallel} \\ B_{out,\perp} &= \mu_0 H_{out,\perp}. \end{aligned} \tag{9}$$

At $\theta = 0$, this leads to

$$B_{out,\parallel} = 0; B_{out,\perp} = B(\mu_0 H_{in}), \tag{10}$$

while at $\theta = \pi/2$,

$$B_{out,\parallel} = \mu_0 H_{in}; B_{out,\perp} = 0. \tag{11}$$

Outside the sphere, its field is a dipole field, for which we know that

$$B_{lateral} = -\frac{1}{2} B_{axis}; \tag{12}$$

in our case, this means that

$$\mu_0 H_{in} = -\frac{1}{2} B(\mu_0 H_{in}) \Longrightarrow \boxed{B(\mu_0 H_{in}) = -2\mu_0 H_{in}}. \tag{13}$$

Note: Since the affirmation in bold might be dubious, I will briefly state how the previous result can be derived in a more formal way. By analogy with the electrostatic case, if we define magnetic charge through

$$\nabla \cdot \mathbf{H} = \rho_m, \tag{14}$$

then the magnetic charge distribution on the surface of the sphere will correspond to two displaced spheres of radius R, magnetic charges $\pm\rho_m$ and displacement $\mathbf{d}$, for which

$$\mathbf{H_{in}} = -\frac{\rho_m \mathbf{d}}{3}. \tag{15}$$

Now, just like in the electrostatic situation,

$$H_{out,\perp} - H_{in,\perp} = \sigma_m, \tag{16}$$

where

$$\sigma_m = \rho_m d \cos\theta. \tag{17}$$

Using this condition at $\theta = 0$, we get

$$H_{out,\perp} - H_{in,\perp} = \rho_m d. \tag{18}$$

But

$$H_{in,\perp} = -\frac{\rho_m d}{3} \Longrightarrow H_{out,\perp} = \frac{2}{3}\rho_m d = -2H_{in}. \tag{19}$$

The $\mathbf{B}$ continuity condition is

$$B_{out,\perp} = B_{in,\perp} \Longrightarrow \mu_0 H_{out,\perp} = B(\mu_0 H_{in}) \Longrightarrow \boxed{B(\mu_0 H_{in}) = -2\mu_0 H_{in}}. \tag{20}$$

Let us draw a line with slope -2 on the $B = B(\mu_0 H)$ graph; this line corresponds to the solution of Eq. (13), and its intersection with the hysteresis curve corresponds to the actual state of the system. The figure is shown on the next page; the intersection points for each of the two temperatures correspond to

- for 20°C, $B_{20} = 0.95$T;
- for 120°C, $B_{120} = 0.48$T.

The magnetization is related to $\mathbf{B}$ and $\mathbf{H}$ through

$$\mathbf{M} = \frac{\mathbf{B} - \mu_0 \mathbf{H}}{\mu_0}; \tag{21}$$

hence, according to Eq. (13),

$$M = -3H, \tag{22}$$

or

$$M = \frac{3}{2}\frac{B}{\mu_0}. \tag{23}$$

The total dipole moment of the sphere is

$$m_m = MV \Longrightarrow m_m = 2\pi R^3 \frac{B}{\mu_0}. \tag{24}$$

Hence, the change in value of the magnetic moment of the sphere due to heating is

$$\Delta m_m = \frac{2\pi R^3}{\mu_0}(B_{120} - B_{20}). \tag{25}$$

Numerically,

$$\boxed{\Delta m_m = -2.35\text{Am}^2}.$$

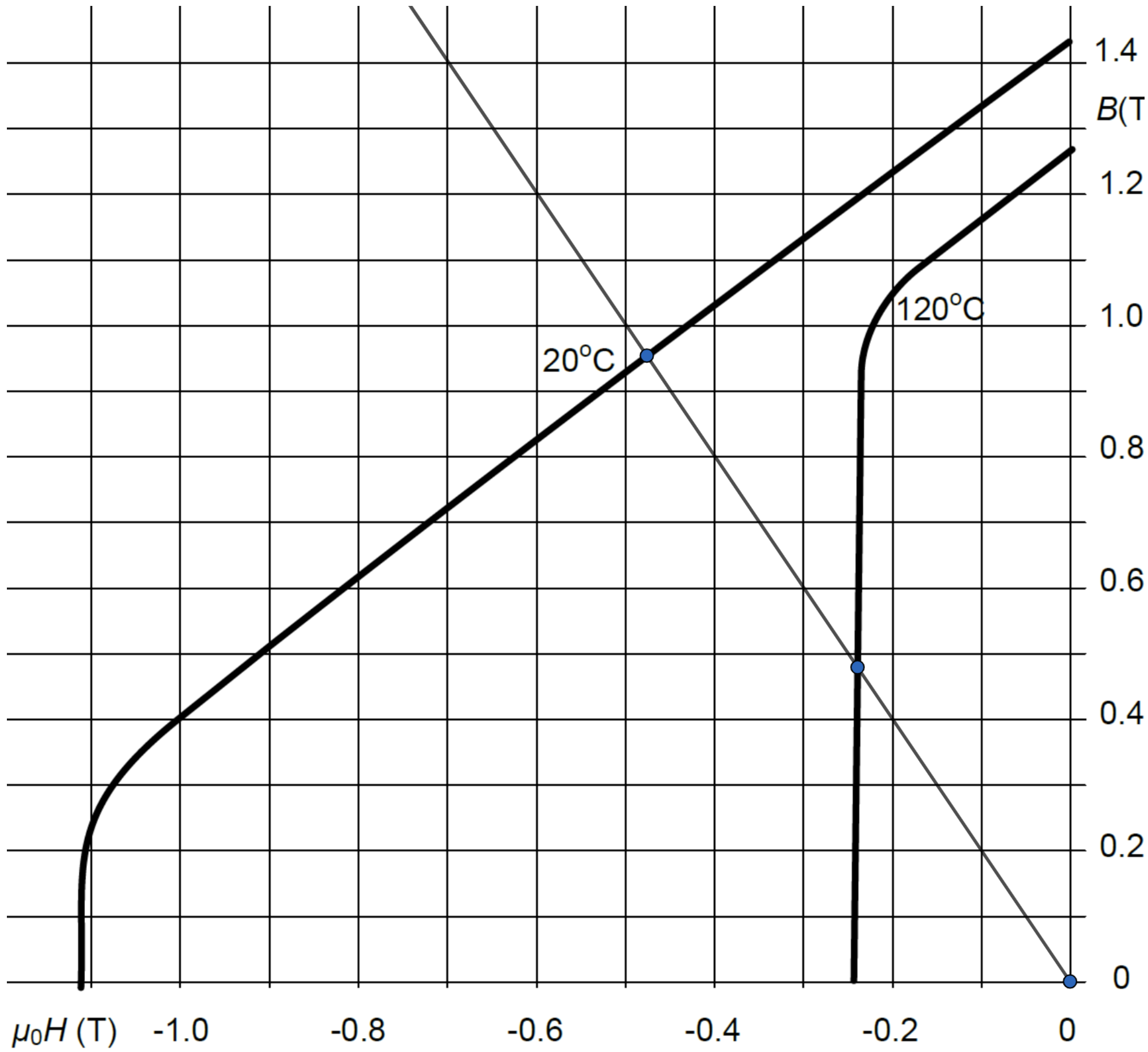

Figure 1: The graph showing the line corresponding to the solution of Eq. (13); the two upper blue points are the actual physical states of the magnet at the two temperatures