Physics Cup P1
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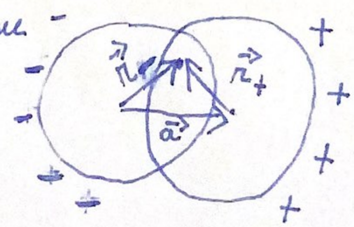
Due to feromagnetic properties of the given material the ball will stay magnetize after removing the magnetic field.
After that the magnetized sphere lies in its own magnetic field, which is created by electric currents on its surface.
We will now prove that magnetic field inside is uniform and how quantities B and H relates to each other:
We will use analogy using 2 uniformly charged ball with charges $\pm\rho$ very small distance from each thus we can treat as only surface charge on a sphere:

From Gauss law we know that $4\pi r^2 \vec{E}_0 = \frac{Q_{enc}}{\varepsilon}\hat{r} = \frac{4}{3}\pi r^3 \frac{\rho}{\varepsilon}\vec{r}$

so $\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{r}$. From fig 1. we can see that for any point in the overlap of the charged ball is given as



$$\vec{E}_{over} = \frac{\rho}{3\varepsilon_0}(r_+ - r_-) = \frac{-\rho}{3\varepsilon_0}\vec{a} = \vec{E}_+ + \vec{E}_-$$

Fig 1.

We recognize $\rho\vec{a}$ to be $\vec{P}$ (polarization vector)
In our case free and bound currents that flow throught sphere
Via Ampere law we know $\nabla \times \vec{H} = \vec{J}_f$ which is analogous to electric field. Analogy then yields that $\varepsilon_0 \leftrightarrow \frac{1}{\mu_0}$ and $\vec{P} \leftrightarrow \vec{M}$ where M is magnetization.
So if $\vec{E} = -\frac{\vec{P}}{3\varepsilon_0}$ then $\vec{H} = -\frac{M}{3}$.

Then the total magnetic field is:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = -\frac{2}{3}\mu_0\vec{M} = -2\mu_0\vec{H}$$

This relationship is purely based on the given shape. In the problem statement we are given hysteresis curve which are related to properties of given material.

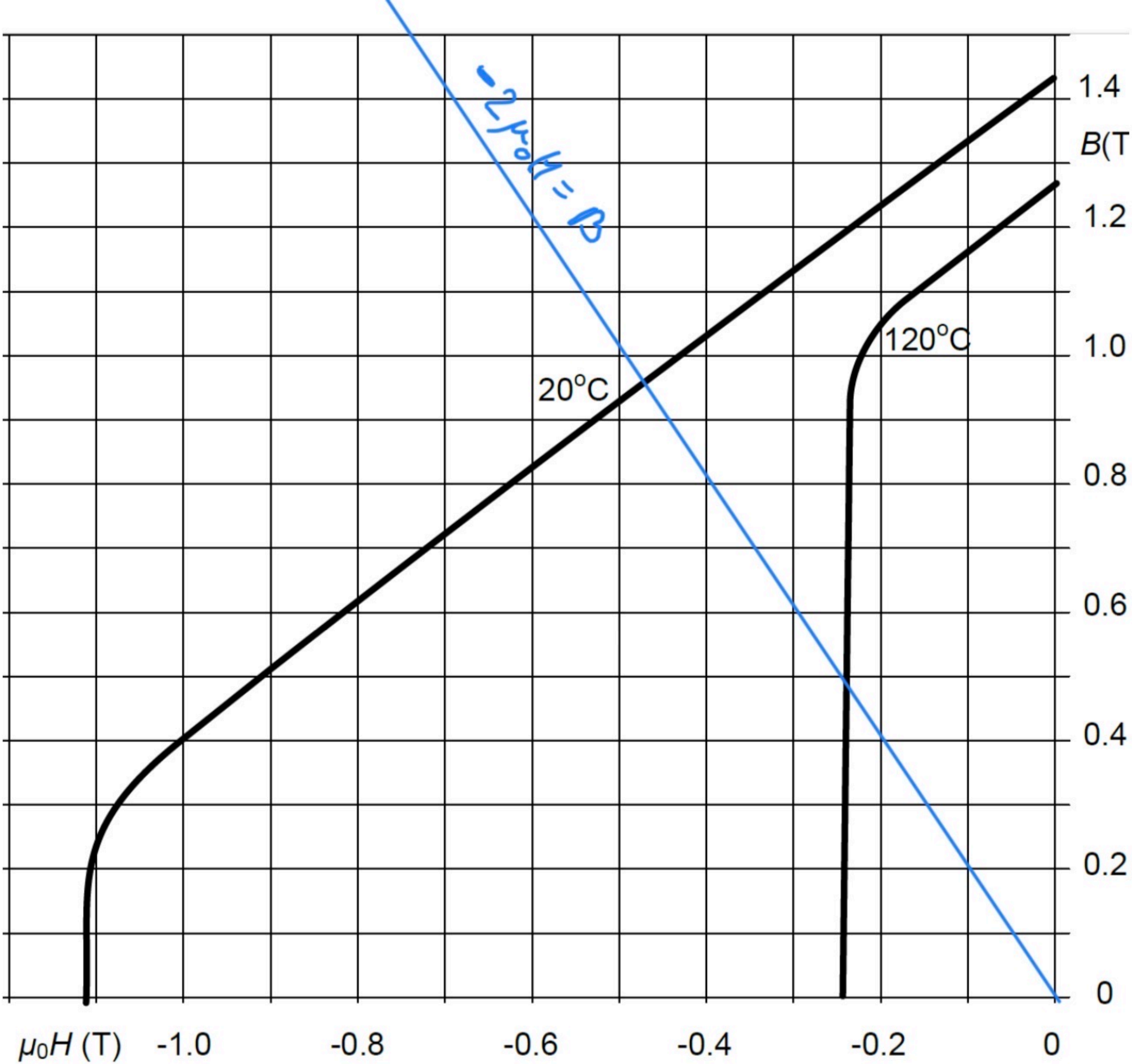
So our ball is located in state defined in the B vs. H plot as intersection of line $-2\mu_0 H = B$ and the hysteresis curve for given temperature.

The coordinates of intersections are aproximately for 20° [-0,47, 0,94] and for 120°C [-0,24; 0,48] so according magnetizations are $\mu_0 M_{20°C} \approx -0{,}47 \cdot (-3) \approx 1{,}41\,T$, $\mu_0 M_{120°C} \approx -3 \cdot (-0{,}24) \approx 0{,}72\,T$

By definition the magnetic dipole moment is given as $\iiint \vec{M}\,dV = \vec{m}$ so in our hommogeous case it is $MV = m$

Change then is $\frac{4}{3}\pi R^3 (M_{20°C} - M_{120°C}) \doteq 2{,}3\ A \cdot m^2$

(The m decreased as expected since heating is very often used to demagnetize objects)



$-2\mu_0 H = B$
1.4
B(T
1.2
1.0
0.8
0.6
0.4
0.2
0
120°C
20°C
μ0H (T)
-1.0
-0.8
-0.6
-0.4
-0.2
0