Problem 2, Physics Cup 2022

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Due to the initial strong and uniform magnetizing field, we can assume the magnetization of the sphere to be constant throughout its volume, and let that be $\mathbf{M}$. Then the dipole moment m of the ball

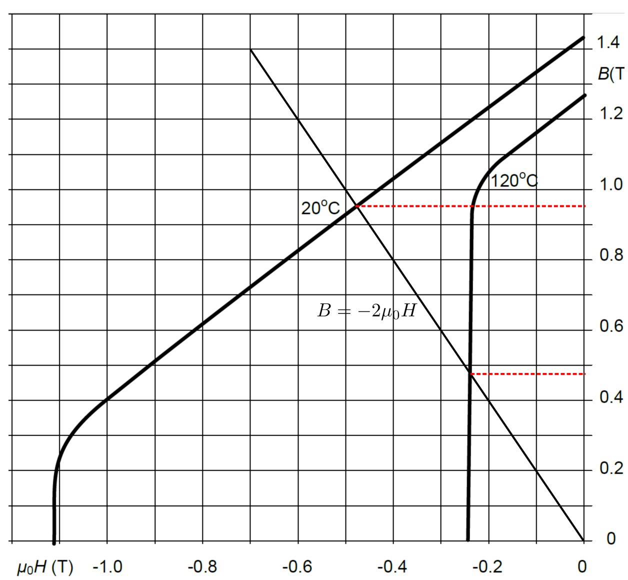
$$\mathbf{m} = \frac{4}{3}\pi R^3 \mathbf{M}$$

The following relation holds true inside an uniformly magnetized sphere

$$\mathbf{B} = -2\mu_0 \mathbf{H} \tag{1}$$

Therefore, $\mu_0 \mathbf{M} = \mathbf{B} - \mu_0 \mathbf{H} = \mathbf{B} + \frac{\mathbf{B}}{2} = \frac{3}{2}\mathbf{B} \implies \Delta M = \frac{3}{2\mu_0}\Delta B$

If we plot equation (1) over the hysteresis graph, the points of intersection with the two hysteresis curves will give us the solution for the respective temperatures.



Thus, if the ball is heated up from 20°C to 120°C, the change in magnetic field $\Delta B \approx 0.475 - 0.95 = -0.475$ and the change in magnetization $\Delta M = \frac{3}{2\mu_0}\Delta B = -5.67 \times 10^5$

Hence, the change of the dipole moment of the ball of radius $R = 0.01$ m

$$\Delta m = \frac{4}{3}\pi R^3 \Delta M = \boxed{-2.38 \text{ A} \cdot \text{m}^2}$$

Proof of the relation (1): The vector potential $\mathbf{A}(\mathbf{r})$ associated with a magnetization $\mathbf{M}(\mathbf{r})$ is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{M}(\mathbf{r}')d\tau' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

where V is the volume of the sphere. Since $\mathbf{M}$ is uniform, it can be taken out of the integral, so that we obtain

$$\mathbf{A}(\mathbf{r}) = \mathbf{M} \times \left[\iiint_V \frac{\mu_0}{4\pi} \frac{(\mathbf{r} - \mathbf{r}')\, d\tau'}{|\mathbf{r} - \mathbf{r}'|^3} \right]$$

The term in brackets can be thought as the electric field $\mathbf{E}$ of a sphere with a uniform charge density equal to $\mu_0 \varepsilon_0$. This electric field can be calculated using Gauss's law:

$$\mathbf{E} = \frac{\mu_0 r}{3} \hat{r}, \text{ for } r < R$$

Therefore $\mathbf{A} = \mathbf{M} \times \mathbf{E}$ equals to

$$\mathbf{A} = \frac{\mu_0 M r}{3} \sin\theta \hat{\phi}, \text{ for } r < R$$

The magnetic field is given by

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{2}{3}\mu_0 \mathbf{M}$$

Finally, using $\mu_0 \mathbf{M} = \mathbf{B} - \mu_0 \mathbf{H}$, we get

$$\mathbf{B} = -2\mu_0 \mathbf{H}$$