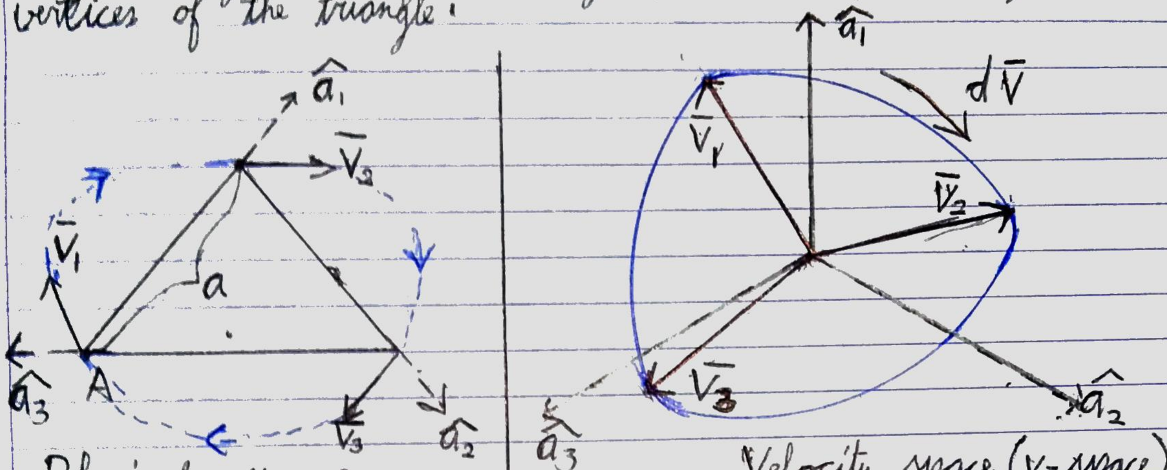
Although the normal force isn't constant in time, the average value of mg can be used because of the negligible stride length.
The maximal acceleration then is μg. For an optimal trajectory the acceleration should be used optimally i.e. the magnitude should always be μg. Clearly the trajectory should pas through the vertices of the triangle.



Physical space

Velocity space (V-space)

$|\bar{V}_1| \geq 0 \wedge |\bar{V}_2| \geq 0 \wedge |\bar{V}_3| \geq 0$. By the symmetry of the problem the critical cases to consider are (1) those in which $|\bar{V}_1| = |\bar{V}_2| = |V_3|$ and the angles between them are equal that is $120°$ and (2) those in which one ore more of $|\bar{V}_1|$, $|\bar{V}_2|$ and $|\bar{V}_3|$ are 0, that is on the boundary of allowed values.

The problem of optimal time corresponds to the problem of minimal perimeter of the loop in V-space; $|d\bar{V}| = dv = \mu g\, dt$ so the time of the (optimal) trajectory is the length of the loop divided by μg.
Case 2 can be eleminated because the loop in V-space will be concave, which clearly is sub-optimal. So case 1 remains.

For the sub-trajectories $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ just one needs to be considered. The others will be the same but just rotated.
Consider $\overrightarrow{AB}$: The distance traveled in the direction of $\hat{a}_1$ in the needed time T is $\int_0^T \bar{V}\cdot\hat{a}_1\, dt = \int_{\bar{V}_0}^{\bar{V}_T} \bar{V}\cdot\hat{a}_1 \frac{dv}{\mu g} = a$ I

The displacement in the direction perpendicular to $\hat{a}_1$ is

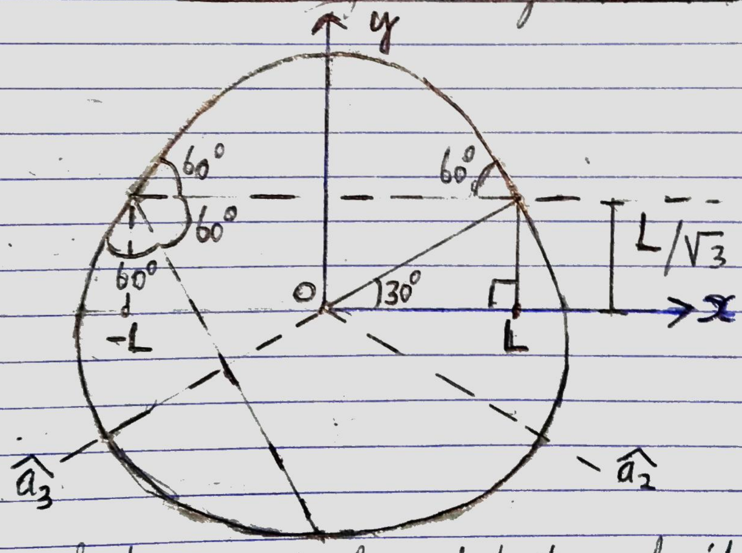
$$\int_{\bar{V}_0}^{\bar{V}_T} (\bar{V} - \bar{V}\cdot\hat{a}_1)\frac{dv}{\mu g} = 0 \quad \text{II}$$

$$T = \int_{\bar{V}_0}^{\bar{V}_T} \frac{dv}{\mu g} \quad \text{III}$$

Let the velocity component in the $\hat{a}_1$ direction $-\bar{V}\cdot\hat{a}_1-$ be y and the one perpendicular to $\hat{a}_1$ to the right be x.
By symmetry y is an even function of x. Hence II is automatically satisfied. The problem then comes to the following:
Minimize T if $\int_{x_1}^{x_f} y\cdot\sqrt{(y')^2+1}\,\frac{dx}{\mu g} = a$ and $T = \int_{x_1}^{x_f} \sqrt{(y')^2+1}\,\frac{dx}{\mu g}$

Minimizing 'T' for a fixed 'a' is equivalent to optimizing 'a' for a fixed 'T'! The problem then is similar to that of finding the shape of a homogeneous rope shaped by constant gravity. Hence, the solution to the problem is an inverted catenary; Inverted to maximize a; a normal catenary would minimize 'a' in the same way in which it minimizes the gravitational potential energy. Therefore $y = -A\cosh\left(\frac{x}{A}\right)+B$ with $A>0$ and $B>0$. (y is even)

Substituting boundary conditions



Since the acceleration can't change instantaneously it should be differentiable. The only angle for which this is the case is an angle of 60°. Hence $y'(L) = -\tan(60°) \Rightarrow -\sinh\left(\frac{L}{A}\right) = -\sqrt{3}$

$\sinh\left(\frac{L}{A}\right) = \sqrt{3}$, $\cosh\left(\frac{L}{A}\right) = 2$ and $\frac{L}{A} = \ln(2+\sqrt{3})$.

Furthermore $y(L) = -A\cosh\left(\frac{L}{A}\right)+B = \frac{L}{\sqrt{3}} \Rightarrow B = \frac{L}{\sqrt{3}} + \frac{2L}{\ln(2+\sqrt{3})}$

Also $\int_{-L}^{L} y\sqrt{(y')^2+1}\,\frac{dx}{\mu g} = a \Rightarrow \int_{-L}^{L}\left(-A\cosh\left(\frac{x}{A}\right)+B\right)\cosh\left(\frac{x}{A}\right)dx = \mu g a$

$$-A^2\cosh\left(\frac{L}{A}\right)\sinh\left(\frac{L}{A}\right)+2AB\sinh\left(\frac{L}{A}\right)-AL = \mu g a$$

$$-\left[\frac{L}{\ln(2+\sqrt{3})}\right]^2\cdot 2\cdot\sqrt{3}+2\frac{L}{\ln(2+\sqrt{3})}\cdot\left(\frac{L}{\sqrt{3}}+\frac{2L}{\ln(2+\sqrt{3})}\right)\cdot\sqrt{3}-\frac{L}{\ln(2+\sqrt{3})}L = \mu g a$$

$$L = \left[\frac{1}{\ln(2+\sqrt{3})}+\frac{2\sqrt{3}}{\ln^2(2+\sqrt{3})}\right]^{-1/2}\left[\mu g a\right]^{1/2}$$

The time needed for $\overrightarrow{AB}$ is $\int_{-L}^{L}\sqrt{(y')^2+1}\,\frac{dx}{\mu g} = \int_{-L}^{L}\cosh\left(\frac{x}{A}\right)\frac{dx}{\mu g} =$

$$\frac{2A}{\mu g}\sinh\left(\frac{L}{A}\right) = \frac{2}{\mu g}\cdot\frac{L}{\ln(2+\sqrt{3})}\cdot\sqrt{3} = \frac{2\sqrt{3}}{\ln(2+\sqrt{3})}\left[\frac{1}{\ln}+\frac{2\sqrt{3}}{\ln^2}\right]^{-1/2}\left[\frac{a}{\mu g}\right]^{1/2}$$

The optimal time for the total trajectory then is

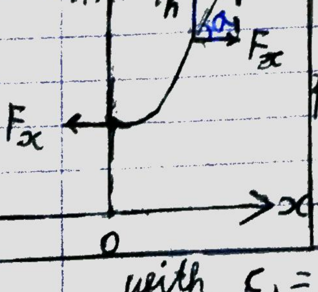
$$6\sqrt{3}\left[\ln(2+\sqrt{3})+2\sqrt{3}\right]^{-1/2}\left[\frac{a}{\mu g}\right]^{1/2}$$

Derivation of the catenary

For a hanging rope of fixed length the gravitational potential energy should be minimal i.e. $\int_{m_i}^{m_f} dm\, g\, h$ is minimal, where dm is a mass differential and h the corresponding height. Suppose the linear mass density of the rope is λ, then dm is the length of dm multiplied by λ: $dm = \lambda\sqrt{(dh)^2+(dx)^2} = \lambda\left[\left(\frac{dh}{dx}\right)^2+1\right]^{1/2}dx$

The potential energy then is $\int_{x_i}^{x_f}\lambda g h\sqrt{(h')^2+1}\,dx$.

A way to find the equation of the catenary is to consider forces:



F_x is the constant horizontal component of the tension (F_t) and F_h is the vertical component, which is equal to the gravitational pull on the mass below h: $F_h = \int_0^x g\lambda\left[(y')^2+1\right]dx$

F_t is tangent to the curve so $h' = \tan\alpha = F_h/F_x \Rightarrow$

$h' = \int (g\lambda/F_x)\left[(h')^2+1\right]^{1/2}dx \Rightarrow dh'/dx = c_1\left((h')^2+1\right)^{1/2}$

$\int dh'\left((h')^2+1\right)^{-1/2} = \int c_1\,dx \Rightarrow \sinh^{-1}(h') = c_1x+c_2 \Rightarrow$

$h' = \sinh(c_1x+c_2) \Rightarrow h = \frac{1}{c_1}\cosh(c_1x+c_2)+c_3$,

with $c_1 = g\lambda/F_x$ and c_2 and c_3 yet to be found.