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1 Introductory remarks

- Let us consider first the side of the satellite facing the Sun. This face may be coloured inhomogeneously. At any rate, the overall heat absorbed by the face depends only on the mean emissivity of the face. Let us call this ε_1. Then, the quantity of heat *absorbed* by this face per unit time will be $\varepsilon_1 AS$. To make the power absorbed maximal, it would be favourable to make ε_1 as large as possible - that is, make the face fully black ($\varepsilon_1 = 1$) - but since it isn't obvious that this leads to maximum useful power output, we will assume that ε_1 can take any value between 0 and 1, and only assume that the colouring is homogeneous. In this case, if the temperature of this face of the plate is T_1, then the heat radiated by this face per unit time will be $\varepsilon_1 A\sigma T_1^4$, so that the overall heat input into the satellite from this face will be

$$\dot{Q}_1 = \varepsilon_1 A(S - \sigma T_1^4). \tag{1}$$

- Similarly, let the mean emissivity of the side not facing the Sun be ε_2, and let its temperature be T_2. Then, the heat lost on this side will be (in absolute value)

$$\dot{Q}_2 = \varepsilon_2 A\sigma T_2^4. \tag{2}$$

- We can consider the system that is formed - composed of two objects, one at temperature T_1, one at T_2, exchanging heat with the surroundings and with each other - as a heat engine, with the useful work that is extracted converted into electrical energy[1]. This quantity of useful work that can be extracted per unit time is precisely the charging power, and is, due to energy conservation, equal to

$$P = \dot{Q}_1 - \dot{Q}_2. \tag{3}$$

- The best possible efficiency of work production is obtained if we use a Carnot heat engine. For such a device, we will have that

$$\frac{\dot{Q}_1}{T_1} = \frac{\dot{Q}_2}{T_2}. \tag{4}$$

Hence, we have to maximize P, under the conditions stated above. But first, let us obtain a simpler expression for it.

2 Obtaining an expression for the power P

Firstly, let us express T_2 in terms of T_1, using Eq. (4):

$$\frac{\varepsilon_1 A(S - \sigma T_1^4)}{T_1} = \frac{\varepsilon_2 A\sigma T_2^4}{T_2} \Longrightarrow T_2^3 = \frac{\varepsilon_1}{\varepsilon_2}\Big(\frac{S}{\sigma T_1} - T_1^3\Big). \tag{5}$$

[1] In the best case scenario - which is the one we will assume to happen in this problem - this transformation can be done with an efficiency of 100%.

Using this, we can express $\dot{Q}_2$ as

$$\dot{Q}_2 = \frac{\varepsilon_1^{\frac{4}{3}}}{\varepsilon_2^{\frac{1}{3}}} A\sigma \left(\frac{S}{\sigma T_1} - T_1^3 \right)^{\frac{4}{3}}, \tag{6}$$

so that, using Eq. (3)

$$P = \varepsilon_1 A(S - \sigma T_1^4) - \frac{\varepsilon_1^{\frac{4}{3}}}{\varepsilon_2^{\frac{1}{3}}} A\sigma \left(\frac{S}{\sigma T_1} - T_1^3 \right)^{\frac{4}{3}}. \tag{7}$$

We see that, assuming known values of ε_1 and ε_2 (along with the other, fixed, parameters), the power is a function of T_1. Hence, let us define x through

$$x = \frac{\sigma T_1^4}{S}. \tag{8}$$

If x were greater than 1, then the face facing the Sun would actually have a net *loss* of heat, which means, according to energy conservation, that either the cold face should *absorb* heat (which is impossible due to the second law of thermodynamics), or we should introduce work into the system (which is not what we want), or both. Hence, $0 < x \leq 1$, and using x, the above expression can be rewritten as

$$\frac{P}{AS} = \varepsilon_1(1-x) - \frac{\varepsilon_1^{\frac{4}{3}}}{\varepsilon_2^{\frac{1}{3}}} \frac{(1-x)^{\frac{4}{3}}}{x^{\frac{1}{3}}}. \tag{9}$$

Now, we will maximize P.

3 Maximizing P

- Firstly, since $0 < x \leq 1$, we see that the second term above - which is substracted - is always positive; hence, to maximize P, we should minimize it. And, as we see that ε_2 appears only in the denominator of this term, it is clear that it would be best to **maximize** it. Hence,

$$\boxed{\varepsilon_2 = 1}$$

- Now, the expression for P becomes

$$\frac{P}{AS} = \varepsilon_1(1-x) - \varepsilon_1^{\frac{4}{3}} \frac{(1-x)^{\frac{4}{3}}}{x^{\frac{1}{3}}}. \tag{10}$$

  Denoting $y = \varepsilon_1(1-x)$, we can rewrite this as

$$\frac{P}{AS} = y - \frac{y^{\frac{4}{3}}}{x^{\frac{1}{3}}}. \tag{11}$$

  We see that for a given value of y, P is maximal if x is maximal. And this means that $1-x$ is minimal, and hence, as $y = \varepsilon_1(1-x)$ is fixed, that ε_1 is **maximal**. Hence, to maximize P, we require that

$$\boxed{\varepsilon_1 = 1}$$

- Further, P can be written as

$$\frac{P}{AS} = 1 - x - \frac{(1-x)^{\frac{4}{3}}}{x^{\frac{1}{3}}} = (1-x)\left(1 - \left(\frac{1}{x} - 1\right)^{\frac{1}{3}}\right). \tag{12}$$

  The derivative of this with respect to x must be 0 - hence,

$$(1-x)\frac{1}{3(1-x)^{\frac{2}{3}} x^{\frac{4}{3}}} - \left(1 - \left(\frac{1}{x} - 1\right)^{\frac{1}{3}}\right) = 0 \Longrightarrow (1-x)^{\frac{1}{3}} \left(\frac{1}{3x^{\frac{4}{3}}} + \frac{1}{x^{\frac{1}{3}}}\right) = 1. \tag{13}$$

- Equation (13) must be solved numerically; its solution is

$$\boxed{x = 0.7506941}$$

- For this value of x, we find that

$$\frac{P}{AS} = 0.07666021, \tag{14}$$

so that, in the end, the maximum power that can be attained is

$$\boxed{P_{max} = 0.07666021AS}$$