Solution to problem No5

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Consider the satellite as a heat engine with the front plate as its (indirect) source and the back plate as its sink. Let Ψ_{front}, Ψ_{back}, Ψ_{el} and Ψ_{sun} denote the heat flux density of respectively the outward-emitted radiation of the front plate, the outward emitted radiation of the back plate, the battery and the sun. Let ϵ_{eff}, T_{front}, T_{back} and σ denote respectively the effective emissivity of the outer side of the back plate, the temperature of the front plate and the back plate and the Stefan-Boltzmann constant. We have the following:
$T_{back} = T$ and $T_{front} = nT$ (with n of course greater than one), $\Psi_{sun} = S$,
$\Psi_{front} = \sigma n^4 T^4$ (evidently the absorptivity and therefore the emissivity should be one), $\Psi_{back} = \sigma \epsilon_{eff} T^4$
and $\Psi_{el} = \Psi_{sun} - \Psi_{front} - \Psi_{back} = S - \sigma(n^4 + \epsilon_{eff})T^4$ (1)
For an optimal Ψ_{el} the process should be reversible and thus the entropy of the satellite is constant, from which it follows that $(\Psi_{sun} - \Psi_{front})/T_{front} = \Psi_{back}/T_{back}$ i.e.
$\sigma(n^4 + \epsilon_{eff} n)T^4 = S$ (2). From (2) and (1) it follows that $\Psi_{el} = \frac{(n-1)\epsilon_{eff}}{n^4+\epsilon_{eff} n} S$.
For $n = 1$, $\Psi_{ell} = 0$ and for $n \to \infty$, $\Psi_{ell} \to 0$. Since Ψ_{ell} is continuous for $n > 1$, for a given ϵ_{eff} Ψ_{ell} is maximal for the unique allowed case where $\frac{\partial \Psi_{el}}{\partial n} = 0$ i.e. $-\frac{3n^4-4n^3-\epsilon_{eff}}{n^2 \cdot \left(n^3+\epsilon_{eff}\right)^2} \epsilon_{eff} = 0 \Rightarrow$ $3n^4 - 4n^3 - \epsilon_{eff} = 0$ Furthermore
$\frac{\partial \Psi_{el}}{\partial \epsilon_{eff}} = \frac{(n-1)n^2}{\left(\epsilon_{eff}+n^3\right)^2} > 0$ for $n > 1$ so Ψ_{el} is maximal for $\epsilon_{eff} = 1$ (as one would expect). We thus have the maximum for $3n^4 - 4n^3 - 1 = 0$ and $n > 1$, that is for

$$n = \frac{\sqrt{9\left(\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}\right)^{\frac{2}{3}} + 4\sqrt[3]{\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}} - 4}}{6\sqrt[6]{\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}}} + \frac{\sqrt{-\sqrt[3]{\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}} + \frac{16\sqrt[6]{\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}}}{9\sqrt{9\left(\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}\right)^{\frac{2}{3}} + 4\sqrt[3]{\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}} - 4}} + \frac{4}{9\sqrt[3]{\frac{2^{\frac{7}{2}}}{27} - \frac{8}{27}}} + \frac{8}{9}}}{2} + \frac{1}{3} \approx 1.4440332233435$$

The maximal constant charging power is $\frac{n-1}{n^4+n} SA \approx 0.0766602099 SA$