Problem No 5.

Let the two plates be at temperatures T_1 (one closer to the Sun), and T_2 such that $T_1 > T_2$. We assume that inside the plate there is a device that converts heat into electrical energy with maximum theoretical efficiency (Carnot's engine) of $\eta = 1 - \frac{T_2}{T_1} = \frac{P}{Q}$, where P is the output power, and Q the input heat power (therefore $P = Q(1 - \frac{T_2}{T_1})$). On the other side, as we are looking for constant P, we have to use the expressions for gray body radiation and make sure the two plate sides are in equilibrium. We label with ϵ_1 and ϵ_2 the two emissivities (Note: the absorptivity is equal to the emissivity) and write the equations for sides 1 and 2 respectively (it doesn't matter if the paint is applied in a pattern, we can still find an effective emissivity ϵ_{eff}).

$$\epsilon_1 S A = \epsilon_1 \sigma A T_1^4 + Q \tag{1}$$

$$\epsilon_2 \sigma A T_2^4 = Q_C \equiv Q - P \tag{2}$$

Dividing (2) by (1) after we put Q on the left we have

$$\frac{\epsilon_2}{\epsilon_1} (\frac{T_2}{T_1})^4 = \frac{Q - Q(1 - \frac{T_2}{T_1})}{\epsilon_1 S A - Q}$$
(3)

Labeling with x the ratio $\frac{T_2}{T_1}$ we have

$$\frac{\epsilon_2}{\epsilon_1}x^4 = \frac{Qx}{\epsilon_1 SA - Q}\tag{4}$$

$$Q = \frac{\epsilon_2 x^4 S A}{x + \frac{\epsilon_2}{\epsilon_1} x^4} \tag{5}$$

From P = Q(1 - x)

$$P = \frac{\epsilon_2 x^3 S A(1-x)}{1 + \frac{\epsilon_2}{\epsilon_1} x^3}$$
(6)

If divided by ϵ_2 we see that the denominator is lowest if both ϵ_1 and ϵ_2 are equal to 1 (the whole plate is therefore painted black; radiation from the sides is neglected as the plate is very small compared to its surface area). Using those values we have to minimize P. There are many ways to do that: because x is between 0 and 1 one can find it by guessing, by setting the derivative to zero and solving the equation (here it will be of a higher order than 2 so Newton's method could be used until 4-significant figures are obtained) or plotting the function.

Graphed in Desmos (looking only at the segment where 0 < x < 1) the highest point on the graph has coordinates (0.6925, 0.0767) so the answer for maximal power is P = 0.0767SA.