Problem 5 Solar Powered Satellite

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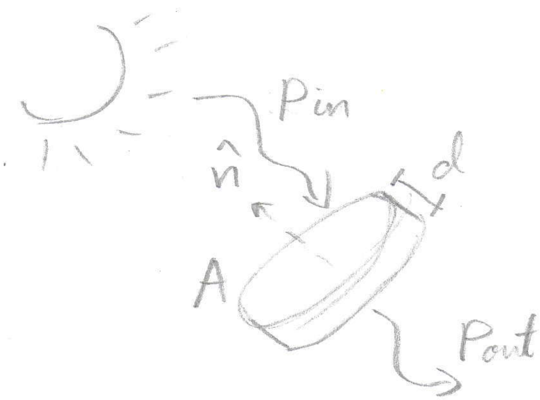
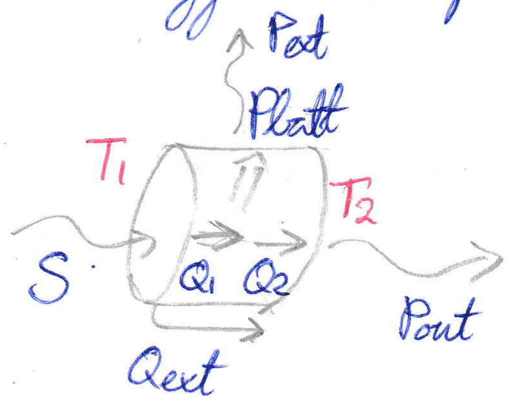


* We need to make the most efficient device using only radiation emission and absorption (and any inner device)

* There is no conduction or convection of the satellite with the space and $T_{out} = 0$ (since we are supposed to neglect CMB)

* There is thermal conduction between parts of the satellite at different temperatures

* To get useful work we will use a Carnot engine inside the satellite



- To use a Carnot engine we need a difference of temperatures between the two faces of the satellite

$$\begin{cases} \varepsilon_1 S A = Q_1 + \varepsilon_1 \sigma A T_1^4 + Q_{ext_1} \\ P_{latt} = Q_1 - Q_2 \\ Q_2 = \varepsilon_2 \sigma A T_2^4 - Q_{ext_2} \\ Q_{ext_1} = P_{ext} + Q_{ext_2} \quad (Q_{ext_1} > Q_{ext_2}) \\ \dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2} \end{cases}$$

heat exchange by conduction in the satellite ($T_1 > T_2$)

$\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$ → Reversible Carnot Engine

$\dfrac{Q_{ext_2}}{T_2} \geq \dfrac{Q_{ext_1}}{T_1}$ → 2° Ther. Law

$$P_{latt} = \varepsilon_1 S A - Q_{ext_1} - \varepsilon_1 \sigma A T_1^4 + Q_{ext_2} - \varepsilon_2 \sigma A T_2^4$$

$$\frac{\varepsilon_1 S A - \varepsilon_1 \sigma A T_1^4 - Q_{ext_1}}{T_1} = \frac{\varepsilon_2 \sigma A T_2^4 - Q_{ext_2}}{T_2}$$

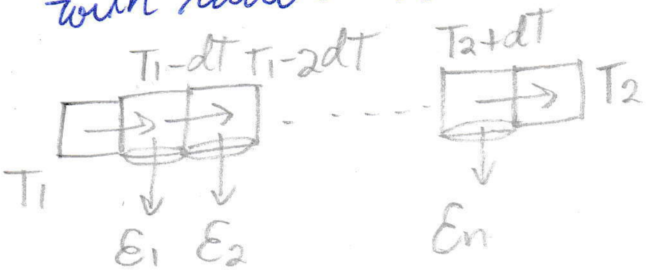
$$\frac{\varepsilon_1 S A}{T_1} + \frac{Q_{ext_2}}{T_2} - \frac{Q_{ext_1}}{T_1} = \varepsilon_2 \sigma A T_2^{\,3} + \varepsilon_1 \sigma A T_1^{\,3}$$

$$P_{batt} = \varepsilon_1 S A - Q_{ext_1} + Q_{ext_2} - \frac{(\varepsilon_1 T_1^4 + \varepsilon_2 T_2^4)}{\varepsilon_2 T_2^3 + \varepsilon_1 T_1^3}\left(\frac{\varepsilon_1 S A}{T_1} + \frac{Q_{ext_2}}{T_2} - \frac{Q_{ext_1}}{T_1}\right)$$

*In order to get the maximum P_{batt} we need to make $Q_{ext_1} = Q_{ext_2} = 0$. Eliminating the thermal conduction with radiation

$$P_{batt} = \varepsilon_1 S A\left(1 - \frac{\varepsilon_1 T_1^4 + \varepsilon_2 T_2^4}{T_1(\varepsilon_2 T_2^3 + \varepsilon_1 T_1^3)}\right)$$

$$P_{batt} = \varepsilon_1 S A\left(\frac{\varepsilon_2 T_1 T_2^3 - \varepsilon_2 T_2^4}{T_1(\varepsilon_2 T_2^3 + \varepsilon_1 T_1^3)}\right)$$



$$P_{batt} = \varepsilon_1 \varepsilon_2 S A\left(\frac{T_1 - T_2}{T_1\left(\varepsilon_2 + \varepsilon_1 \frac{T_1^3}{T_2^3}\right)}\right)$$

*Making $Q_{ext_1} = Q_{ext_2} \approx 0$ through a gradual change of temperature and emissivity (color) achieving thermo-dynamical equilibrium.

$\rightarrow x = \frac{T_1}{T_2}$

$\rightarrow$ $$\boxed{P_{batt} = \varepsilon_1 \varepsilon_2 S A\left(\frac{x-1}{x(\varepsilon_2 + \varepsilon_1 x^3)}\right)}$$

To maximize $P_{batt} \rightarrow \varepsilon_1 = \varepsilon_2 = 1$

$$P_{batt} = S A\left(\frac{x-1}{x(1+x^3)}\right)$$

$$\frac{dP_{batt}}{dx} = \frac{1}{x(1+x^3)} - \frac{x-1}{[x(1+x^3)]^2}\left(1 + x^3 + x(3x^2)\right) = 0$$

$$3x^4 - 4x^3 - 1 = 0$$

$$x(1+x^3) = (x-1)(1+4x^3)$$

$$x + x^4 = x + 4x^4 - 1 - 4x^3$$

$\rightarrow$ $\boxed{x \approx 1.4440}$

$$P_{latt\ máx} = SA\left(\frac{x-1}{x(1+x^3)}\right) \longrightarrow x = 1.4440$$

$$\boxed{P_{latt\ máx} = 0.07666\ SA}$$

Which means that the maximum efficiency of the satellite device is 7.666 %