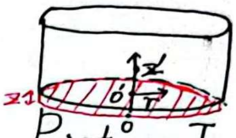
Solution:

Let the distance between its bottom face and the bottom of the water container is $z(t)$ and r, z in Picture I



Picture I

Because of Kelvin theorem on circulation,

we have $\oint_c \vec{v}\cdot d\vec{l}=0$ in full space. $\therefore \nabla\times\vec{v}=0$

For the water, we have $\nabla\cdot\vec{v}=0$

$\because \rho_c>\rho_w \quad 10\rho_c h\ll\rho_w \quad \therefore R\gg h$

$\therefore \frac{dz}{dt}\cdot\pi r^2=2\pi r z\, v(r,z)$

$\therefore \vec{v}(r,z)=\frac{\vec{r}}{2z}\frac{dz}{dt}$

We have Newton's second law in the water,

$\rho_w\frac{d\vec{v}}{dt}=\vec{f}-\nabla p \quad \therefore \rho_w\frac{\partial\vec{v}}{\partial t}+\rho_w(\vec{v}\cdot\nabla)\vec{v}+\nabla p=\vec{f}$

$\because (\vec{v}\cdot\nabla)\vec{v}=\nabla\left(\frac{v^2}{2}\right)-\vec{v}\times(\nabla\times\vec{v})=\nabla\left(\frac{v^2}{2}\right)$

$\therefore \rho_w\frac{\partial\vec{v}}{\partial t}+\rho_w\nabla\left(\frac{v^2}{2}\right)+\nabla p=\vec{f}=-\rho_w g\vec{z}$

$\because z(t)$ isn't connected with r

$\therefore \frac{\rho_w\vec{r}}{2}\left[\frac{d^2z}{dt^2}\frac{1}{z}-\frac{1}{z^2}\left(\frac{dz}{dt}\right)^2\right]+\frac{\rho_w}{2}\left(\frac{\frac{dz}{dt}}{2z}\right)^2\cdot 2\vec{r}+\nabla p=-\rho_w g\vec{z}$

To the water in red, we can neglect z'

$\therefore \nabla p=-\frac{\rho_w\vec{r}}{2}\frac{1}{z}\frac{d^2z}{dt^2}+\frac{\rho_w}{4}\left(\frac{dz}{dt}\frac{1}{z}\right)^2\vec{r}$

$\therefore p(r)=\rho_w gH-\left[\frac{\rho_w}{2z}\frac{d^2z}{dt^2}-\frac{\rho_w}{4}\left(\frac{dz}{dt}z\right)^2\right]\frac{r^2}{2}$

$\therefore F_{\text{water to object}\,1}=\int_0^R p(r)\cdot 2\pi r\,dr=\rho_w gH\pi R^2-\left(\frac{\rho_w}{2z}\frac{d^2z}{dt^2}-\frac{\rho_w}{4}\left(\frac{dz}{dt}\frac{1}{z}\right)^2\right)\frac{\pi R^4}{4}$

We have Newton's sencond law,

$-\rho_c gH\pi R^2+F_{\text{water to object}}=\rho_c H\pi R^2\frac{d^2z}{dt^2}$

Let $\left(\frac{dz}{dt}\right)^2=y$, then

$(\rho_w-\rho_c)gH\pi R^2=\frac{1}{2}\frac{dy}{dz}\left(\rho_cH\pi R^2+\frac{\rho_w}{2z}\frac{\pi R^4}{4}\right)-\frac{y}{z^2}\frac{\rho_w}{4}\frac{\pi R^4}{4}$

$(\rho_w-\rho_c)gH=\frac{1}{2}\frac{dy}{dz}\left(\rho_cH+\frac{\rho_w}{8z}R^2\right)-\frac{1}{16}\rho_w\frac{y}{z^2}R^2$

$\because \rho_w R^2>\rho_w RH\gg 10\rho_c hH>8\rho_c Hh>8\rho_c Hz$

$\therefore \frac{dy}{dz}-\frac{y}{z}=(\rho_w-\rho_c)gH\frac{16z}{\rho_w R^2}$

$\therefore y(z)=z\left(\int(\rho_w-\rho_c)\frac{16gH}{\rho_w R^2}\,dz+C\right)$

$\because y(h)=0$

$\therefore y(z)=z\,\frac{\rho_c-\rho_w}{\rho_w R^2}16gH(h-z)=\left(\frac{dz}{dt}\right)^2$

$\therefore t=\int_0^h\frac{dz}{\sqrt{y(z)}}=\sqrt{\frac{\rho_w R^2}{16(\rho_c-\rho_w)gH}}\int_0^1\frac{dx}{\sqrt{x-x^2}}=\frac{\pi}{4}\sqrt{\frac{\rho_w}{\rho_c-\rho_w}}\frac{R}{\sqrt{gH}}$

t isn't connected with h when $h\ll R$