10th PHYSICS CUP

Problem No.2

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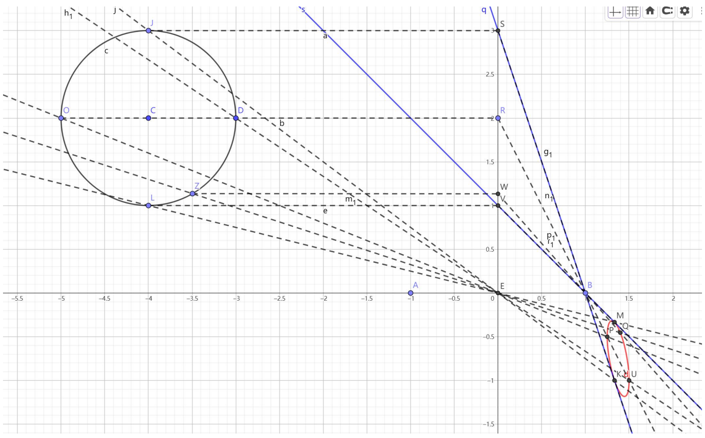
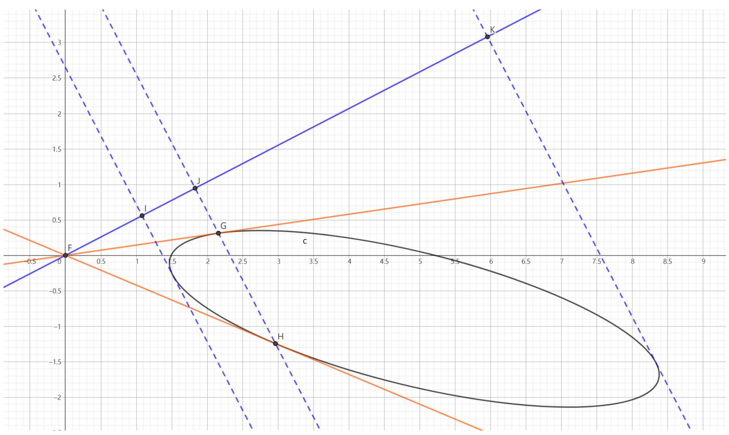
In order to find out the possible relationship between the image and the focus more quickly, we can try to draw a circle through a thin lens first.

（The lens in the picture is at the origin, the optical axis is the x-axis, and the focus is AB.）



When we focus on the tangent of the ellipse, we can find that the tangent comes from the upper and lower points of the circle. So we can draw a conclusion that the connection between the focus at the end of the image and the two tangent points of the image is perpendicular to the optical axis of the lens.

So we can determine the position of the optical axis in the topic, and we know that the image closest to the lens comes from the object farthest from the lens, and the image farthest from the lens comes from the thing closest to the lens. So we know that the four points on the ellipse originate from four special positions of the object, so we can get the equation:



The (1)(2)(3) equations come from the lens imaging formula, and the (4) comes from the similar triangle between the object and the image, Where u is the object distance, f is the focal length, and r is the radius of the object. (Note: through the calculation, we can know that (1)(2)(3) is not independent.)

$$\begin{cases} \dfrac{1}{u+r}+\dfrac{1}{\overline{FI}+f}=\dfrac{1}{f} & (1) \\ \dfrac{1}{u}+\dfrac{1}{\overline{FJ}+f}=\dfrac{1}{f} & (2) \\ \dfrac{1}{u-r}+\dfrac{1}{\overline{FK}+f}=\dfrac{1}{f} & (3) \\ \dfrac{2r}{u}=\dfrac{\overline{GH}}{uf/(u-f)} & (4) \end{cases}$$

We can use three of these equations (Any two of (1)(2)(3) and (4), The following solution takes (1)(2)(4) as an example) to solve the expression of f.

$$f=\frac{\overline{FI}\cdot\overline{GH}}{2\left(\overline{FJ}-\overline{FI}\right)}=\frac{\overline{FI}\cdot\overline{GH}}{2\overline{JI}}$$

Using different combinations, you can get the following three results.

$$f=\frac{\overline{FI}\cdot\overline{GH}}{2\overline{IJ}}\text{or}\frac{\overline{FK}\cdot\overline{GH}}{2\overline{JK}}\text{or}\frac{\left(\overline{FI}+\overline{FK}\right)\cdot\overline{GH}}{2\overline{IK}}$$

Taking the first result as an example, the lens position O can be obtained.
(NMIF and IGHL are parallelograms and MLGF is a pair of similar triangles.)