Physics Cup, Problem Number 2: Image of a circle.

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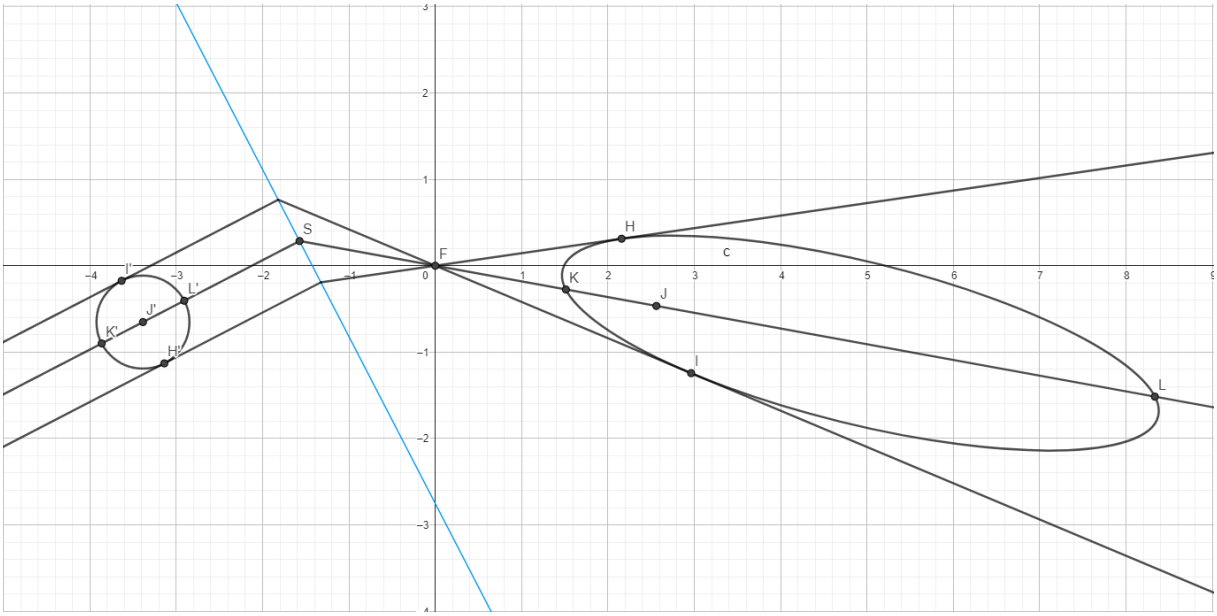
Contents:

Constructing the center of the lens:

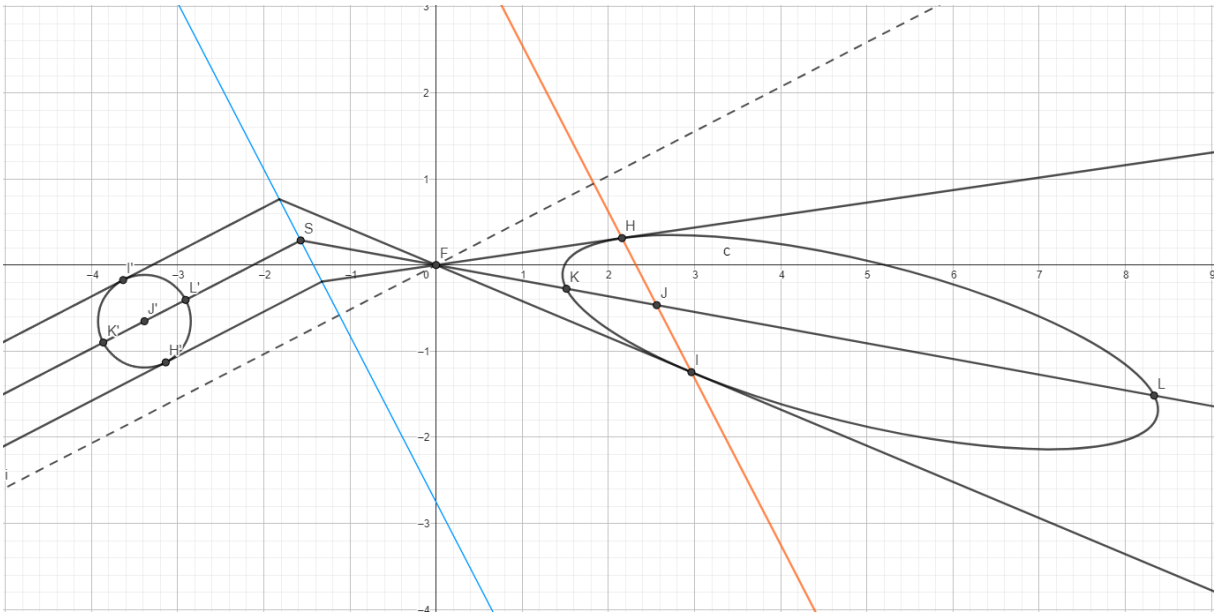
Finding the images of the defining points of the circle:

In order to successfully construct the center of the lens the images of the defining points of the circle had to be found first. Namely points H, I, J, K, L, which in turn are the images of the following points on the circle: points closest (H') and furthest (I') from the optical axis, circle's center (J'), points furthest (K') and closest (L') to the lens. Since light beams running orthogonally to the lens intersect at the focal point, points H and I were constructed as intercepts of the ellipse and lines tangent to the ellipse which are passing through the focal point F. Then point J was defined as the midpoint of H and I. Subsequently the line which passes through the focal point F and point J was drawn and used to define points K and L as its intercepts with the ellipse.



Defining the optical axis:

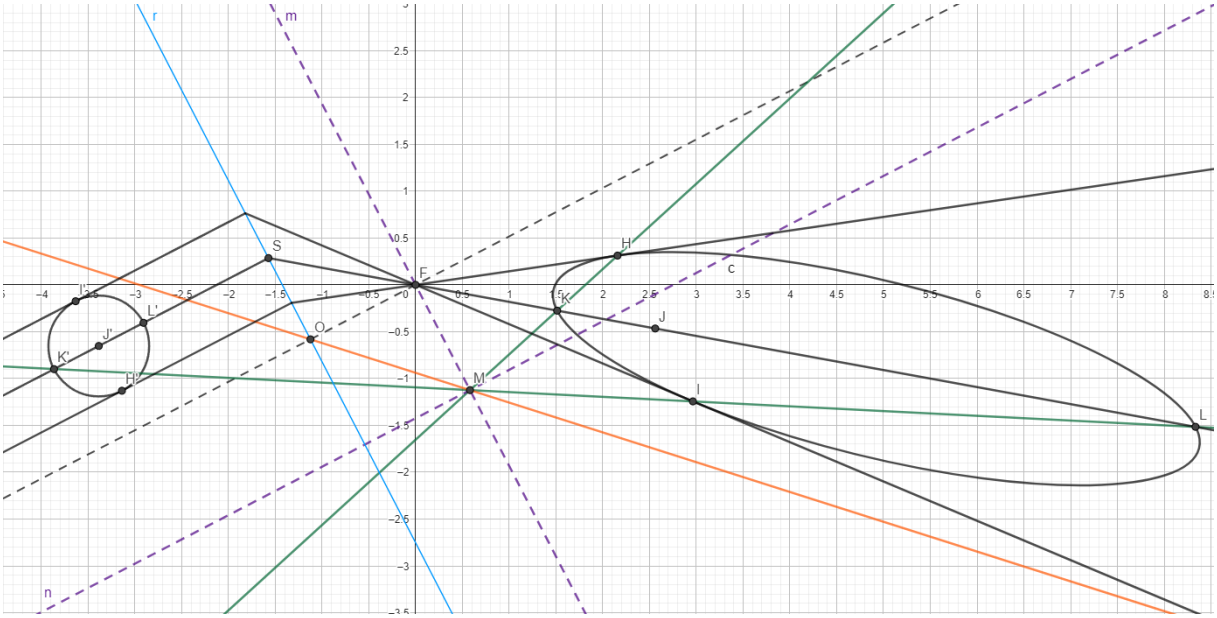
After finding points H, I, J, K, L line which passes through points H, J, I was constructed, and the optical axis was defined as the line which passes through the focal point F and is orthogonal to the newly constructed line.



Defining the center of the lens:

Constructing the center O of the circle was possible due to the fact that parallel beams after passing through the lens intersect at one point on the plane and since it is also known that the line which passes through points I', L' and the line which passes through points K', H' are at an angle of $\frac{\pi}{4}$ both to the lens and the optical axis (it is due to the fact that these points were carefully chosen in the first paragraph). Based on that, lines passing through points H, K and through I, L and their intercept M were defined. Then two additional lines n, m which are in turn parallel and perpendicular to the optical axis and are both passing through point M where drawn. Finally, by constructing the bisector of the angle between lines n and m the point O was found as an intercept between the bisector and the optical axis. The coordinates of the center of the lens were found:

$$O = (-1.12358, -0.5813210001).$$



Remarks:

Initially the whole construction was drawn using lines instead of rays, only after finding the center of the lens these were additionally constructed to clarify the drawing.