Amendment to Physics Cup 2025 Problem 1 Solution

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Previously I used energy of the image charge system, and calculated the forces. However, upon attempt on the summation of forces directly, it turns out that the d^4 term with 0th order R^2 term cancels. Hence, this reveals the need to include the 1st order of R^2 term. My final answer is

Answer: $F = -16\pi\varepsilon_0 \frac{\omega^2 B^2 R^{13}}{L^9}$ or $-16\pi\varepsilon_0 \frac{R^{13}}{L^9}\left(\omega B - \frac{m_e\omega^2}{e}\right)^2$

Solution: Continuing from previous solution (I shifted it below)

$$F = -kq^2R\left(\frac{2L}{((L-d)^2-R^2)^2} + \frac{L+d}{((L+d)^2-R^2)^2} + \frac{4L}{(L^2-R^2)^2} - \frac{4L-2d}{((L-d)L-R^2)^2} - \frac{4L+2d}{((L+d)L-R^2)^2} + \frac{2L}{(L^2-d^2-R^2)^2}\right) - \frac{k2q^2Rd^2}{L(L^2-d^2)}\left(\frac{1}{(L-d)^2} + \frac{1}{(L+d)^2} - \frac{2}{L^2}\right)$$

$$F \approx -144\frac{kq^2d^4R^3}{L^9} = -16\pi\varepsilon_0\frac{R^{13}}{L^9}A^2$$

Where $A = \omega B - \frac{m_e\omega^2}{e} \approx \omega B$.

Previous Physics Cup 2025 Problem 1 Solution

Sim Hui Xiang

Problem: The center of a non-magnetic metal ball with radius *R* is at the origin; the ball is in a homogeneous magnetic field *B* that is parallel to the x-axis. The ball rotates with an angular speed ω around the x-axis. At *x*=*L*, y=z=0, there is another identical metal ball that does not rotate. Find the interaction force between the two balls, assuming that $L \gg R$ and $R \ll \sqrt{\rho/\mu\omega}$, where ρ denotes the ball's resistivity and μ is the permeability.

Answer: $F = -8\pi\varepsilon_0 \frac{\omega^2 B^2 R^{11}}{L^7}$ or $-8\pi\varepsilon_0 \frac{R^{11}}{L^7}\left(\omega B - \frac{m_e\omega^2}{e}\right)^2$

Solution: To not dissipate energy, when the ball spins, the electrons must move with the ball. Hence, the velocity of the electron is

$$\vec{v} = \vec{\omega} \times \vec{s}$$

where $\vec{s}$ denotes the displacement from the rotating axis.

Hence, Newton's law reads

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = -m_e s\omega^2 \hat{s}$$

This gives the electric field induced in the ball as

$$\vec{E} = -s\left(\omega B - \frac{m_e\omega^2}{e}\right)\hat{s} \equiv -sA\hat{s}$$

Let us denote $A = \omega B - \frac{m_e\omega^2}{e}$. If $\frac{\omega}{B} \ll \frac{e}{m_e}$, which is usually the case, then $A \approx \omega B$. The potential at the surface of the ball is

$$\vec{E} = -\nabla V = -sA\hat{s}$$

$$V = \frac{1}{2}As^2 + c$$

Here, c is an arbitrary constant.

We seek to find an image charge that resembles this potential profile at the surface. In particular, the potential is a function of $\cos^2\theta$ at the surface

$$V = \frac{1}{2}A R^2 \sin^2\theta + c_1 = -\frac{1}{2}A R^2 \cos^2\theta + c_2$$

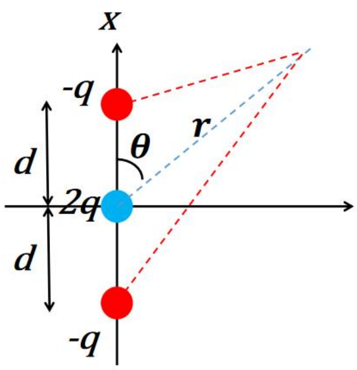

Diagram of a Linear Quadrupole

It turns out that a linear quadrupole has the desired potential. In particular, the one shown on the left has the potential as a function of θ as

$$V(r,\theta) = \frac{qd^2}{4\pi\varepsilon_0 r^3}(1 - 3\cos^2\theta)$$

Equating this to the above yields

$$qd^2 = \frac{2\pi\varepsilon_0}{3}AR^5$$

Hence, we see that the problem have reduced to simply finding the interaction between a linear quadrupole to a metal ball, radius R, at a distance L away.

The interaction between a point charge and a metal ball can be solved by the method of image charge. A linear quadrupole is essentially 3 point charges, placed at different places.

In particular, if a point charge, q, is placed at a distance L from the center of a metal ball, radius R, its image charge is at a distance x = $\frac{R^2}{L}$ from the center and the charge is $q' = -q\frac{R}{L}$. Hence, the image charge of a quadrupole, at a distance L has respective image charges

Index	Original charge	Original Distance	Image Charge	Image distance
1	$-q$	$L-d$	$\frac{qR}{L-d}$	$\frac{R^2}{L-d}$
2	$2q$	L	$-\frac{2qR}{L}$	$\frac{R^2}{L}$
3	$-q$	$L+d$	$\frac{qR}{L+d}$	$\frac{R^2}{L+d}$

We can find the potential energy, U, between the image system and the real system and take derivative to get the force.

$$F = -\frac{1}{2}\nabla U$$

Take half because it is the potential energy from image charge systems. Alternatively we can find the force directly by summation of kqq'/r^2. Nevertheless, I did the former approach. U is given to be

$$U = -kq^2R\left(\frac{1}{(L-d)^2-R^2}+\frac{1}{(L+d)^2-R^2}+\frac{4}{L^2-R^2}-\frac{4}{(L-d)L-R^2}-\frac{4}{(L+d)L-R^2}+\frac{2}{(L-d)(L+d)-R^2}-\frac{4d^4}{L^2(L^2-d^2)^2}\right)$$

Here k = $\frac{1}{4\pi\varepsilon_0}$. Expanding U to the order of d^4 term, and using $L \gg R$, we get

$$U \approx -\frac{24kq^2d^4R}{L^6} \quad \Rightarrow F \approx -\frac{72kq^2d^4R}{L^7}$$

Sub $qd^2 = \frac{2\pi\varepsilon_0}{3}AR^5$ yields,

$$F \approx -\frac{8\pi\varepsilon_0 R^{11}}{L^7}A^2;$$

Where $A = \omega B - \frac{m_e\omega^2}{e} \approx \omega B$.