

1 General Analysis

Initially, we know that since $\sigma \gg \rho d^2 R \Omega^2$:

Since this system does not generate vorticity (Kelvin's theorem), we can define a velocity potential ϕ :

$$\vec{v} = \nabla \phi \quad (1)$$

Since the fluid can be considered incompressible, and given the boundary conditions:

$$\nabla \cdot \vec{v} = \nabla^2 \phi = 0 \quad (2)$$

$$\left. \frac{\partial \phi}{\partial n} \right|_{\text{except holes}} = 0 \quad (3)$$

By analogy with the electric field generated by infinitely long straight charged wires: on a circle, if there are charged wires with opposite charges, the electric field generated at the circular boundary is parallel to the circle. Moreover, our incompressibility condition makes this condition naturally satisfied.

Therefore, we assume $\vec{r}_i = R(\cos \theta_i, \sin \theta_i)$, and assume (applying periodic conditions):

$$\vec{v} = \sum_{i=1}^6 \frac{\phi_i (\vec{r} - \vec{r}_i)}{(\vec{r} - \vec{r}_i)^2} \quad (4)$$

$$\sum_{i=1}^6 \phi_i = 0 \quad (5)$$

2 Without Baffles

To solve for the outflow mode, we need to find the scenario most likely to occur in the system (minimum effective mass).

The kinetic energy of the system is:

$$E_k = \int dS \frac{1}{2} \rho d \vec{v}^2 \quad (6)$$

where

$$\vec{v}^2 = \sum_{ij} \frac{\phi_i \phi_j (\vec{r} - \vec{r}_i) \cdot (\vec{r} - \vec{r}_j)}{(\vec{r} - \vec{r}_i)^2 (\vec{r} - \vec{r}_j)^2} \quad (7)$$

Define $f_{ij} = \int_0^R r dr \int_0^{2\pi} d\theta \frac{(\vec{r} - \vec{r}_i) \cdot (\vec{r} - \vec{r}_j)}{(\vec{r} - \vec{r}_i)^2 (\vec{r} - \vec{r}_j)^2}$. To calculate this, let $z_i = R e^{i\theta_i}$, $z = r e^{i\theta}$:

$$\int_0^{2\pi} d\theta \frac{(\vec{r} - \vec{r}_i) \cdot (\vec{r} - \vec{r}_j)}{(\vec{r} - \vec{r}_i)^2 (\vec{r} - \vec{r}_j)^2} = \oint_{|z|=r} \frac{dz}{iz} \Re \left[\frac{1}{(z - z_i)(z^* - z_j^*)} \right] = \Re \left[\oint_{|z|=r} \frac{dz}{iz} \frac{1}{(z - z_i)(z^* - z_j^*)} \right] \quad (8)$$

Applying the Residue Theorem:

$$\begin{aligned} \oint_{|z|=r} \frac{dz}{iz} \frac{1}{(z - z_i)(z^* - z_j^*)} &= \oint_{|z|=r} \frac{dz}{i} \frac{1}{(z - z_i)(r^2 - z_j^* z)} = -2\pi \frac{1}{r^2 - z_i z_j^*} \\ &= -2\pi \frac{r^2 - R^2 \cos(\theta_i - \theta_j) + i R^2 \sin(\theta_i - \theta_j)}{r^4 + R^4 - 2r^2 R^2 \cos(\theta_i - \theta_j)} \end{aligned} \quad (9)$$

Direct calculation yields:

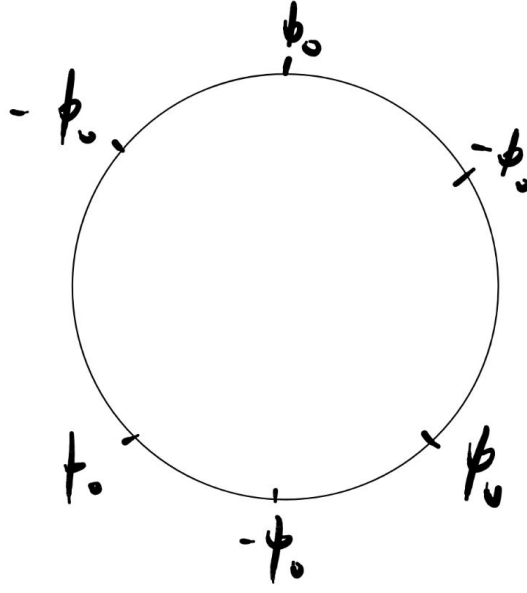
$$f_{ij} = -\frac{\pi}{2} \ln[2(1 - \cos(\theta_i - \theta_j))] \quad (10)$$

Thus, the kinetic energy is obtained as:

$$E_k = -\frac{\pi}{4} \rho d \sum_{ij} \phi_i \phi_j \ln[2(1 - \cos(\theta_i - \theta_j))] \quad (11)$$

Under the constraint $\sum_i \phi_i = 0$, we obtain:

$$\frac{\delta}{\delta \phi_i} \sum_{lp} \phi_l \phi_p \ln[2(1 - \cos(\theta_l - \theta_p))] + \lambda = 0 \quad (12)$$



Therefore, we get:

$$\phi_{i+2} = \phi_{i-2} \quad (13)$$

Since i is arbitrary and can be changed freely, we observe that at the minimum:

$$\phi_1 = -\phi_2 = \phi_3 = -\phi_4 = \phi_5 = -\phi_6 = \phi_0 \quad (14)$$

And assuming the displacement at the i -th hole is x_i , satisfying:

$$\dot{x}_i = -\alpha \frac{\phi_i}{d} \quad (15)$$

The kinetic energy is (denoting the divergent term as $-I$):

$$E_k = -\frac{\pi}{4\alpha^2} \rho d^3 \dot{x}^2 [-6I + \ln 3 \cdot (6) + 2 \ln 2 \cdot (-3)] \quad (16)$$

The potential energy under the action of atmospheric pressure and centrifugal force is:

$$E_p = \sum_i P_0 d^2 x_i - \frac{1}{2} \Omega^2 \rho \int_0^x d^2 (R+t)^2 dt \approx - \sum_i \frac{1}{2} \Omega^2 \rho d^2 R x_i^2 = -3 \Omega^2 \rho d^2 R x^2 \quad (17)$$

So the energy is:

$$E = \frac{\pi}{4\alpha^2} \rho d^3 \dot{x}^2 [6I - 6 \ln 3 + 6 \ln 2] - 3 \Omega^2 \rho d^2 R x^2 \quad (18)$$

The resulting equation of motion is:

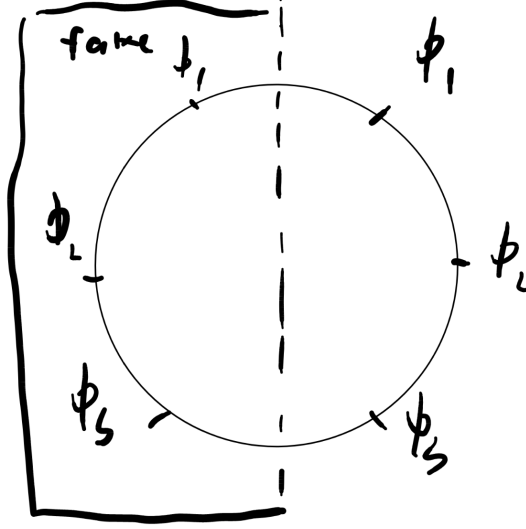
$$\ddot{x} = \frac{2\alpha^2 \Omega^2 R}{\pi d (I - \ln 3 + \ln 2)} x \quad (19)$$

Therefore:

$$x(t) = x e^{\sqrt{\frac{2\alpha^2 \Omega^2 R}{\pi d (I - \ln 3 + \ln 2)}} t} \quad (20)$$

Correspondingly:

$$T_1^2 = (\ln 2)^2 \frac{\pi d (I - \ln 3 + \ln 2)}{2\alpha^2 \Omega^2 R} \quad (21)$$



3 With Baffles

At this point, we only consider the right half. The fictitious region on the left contains image sources. The magnitude of the sources at positions symmetric to the baffle should be consistent with the real sources, thereby making the streamlines at the baffle parallel to the baffle.

Finding the minimum equivalent mass:

$$E_k = -\frac{\pi}{4}\rho d\frac{1}{2}[-2I(\phi_1^2 + \phi_2^2 + \phi_3^2) + \ln 3(\phi_1\phi_2 + \phi_2\phi_3 + \phi_3\phi_1) + 2\ln 2(2\phi_1\phi_3 + \phi_2^2)] \quad (22)$$

$$\phi_1 + \phi_2 + \phi_3 = 0 \quad (23)$$

It is easy to obtain:

$$\phi_1 = \phi_3 = -\frac{1}{2}\phi_2 = -\frac{1}{2}\phi_0 \quad (24)$$

Thus:

$$E_k = \frac{3\pi}{8}\rho d^3\alpha^2\dot{x}^2[I + \frac{1}{2}\ln 3 - \ln 2] \quad (25)$$

$$E_p = -\frac{1}{2}\Omega^2\rho d^2R[x^2 + (\frac{-x}{2})^2 \times 2] = -\frac{3}{4}\Omega^2\rho d^2Rx^2 \quad (26)$$

Similarly:

$$x(t) = xe^{\sqrt{\frac{2\alpha^2\Omega^2R}{\pi d(I + \frac{1}{2}\ln 3 - \ln 2)}}t} \quad (27)$$

$$T_2^2 = (\ln 2)^2 \frac{\pi d(I + \frac{1}{2}\ln 3 - \ln 2)}{2\alpha^2\Omega^2R} \quad (28)$$

Finally, we get:

$$T_2^2 - T_1^2 = \frac{\pi d(\ln 2)^2}{2\alpha^2\Omega^2R}(\frac{3}{2}\ln 3 - 2\ln 2) \quad (29)$$