

Physics Cup 2026 – Problem #1

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Problem 1 Define $a \equiv F/m$. Relative slip velocities at the two faces are

$$\vec{r}_1 = \vec{u}, \quad \vec{r}_2 = \vec{u} - \vec{v}.$$

Each friction force opposes the corresponding slip, hence the brick's acceleration is

$$\dot{\vec{u}} = -a \left(\frac{\vec{r}_1}{|\vec{r}_1|} + \frac{\vec{r}_2}{|\vec{r}_2|} \right). \quad (1)$$

Two invariants and a useful parametrization. Let $p = |\vec{r}_1|$, $q = |\vec{r}_2|$, and denote

$$s = p + q, \quad D = p - q.$$

From (1),

$$\frac{d}{dt}(p - q) = \frac{d}{dt}|\vec{r}_1| - \frac{d}{dt}|\vec{r}_2| = -a(1 + \cos \varphi) + a(1 + \cos \varphi) = 0,$$

so D is a constant. At $t = 0$, $p(0) = u_0$, $q(0) = \sqrt{u_0^2 + v^2} \equiv R$, hence

$$D = u_0 - R \quad (\text{constant}). \quad (2)$$

Moreover,

$$\frac{ds}{dt} = \frac{dp}{dt} + \frac{dq}{dt} = -2a(1 + \cos \varphi), \quad \cos \varphi = \frac{p^2 + q^2 - v^2}{2pq}. \quad (3)$$

Using $pq = \frac{1}{4}(s^2 - D^2)$ and $p^2 + q^2 = (s^2 + D^2)/2$ gives

$$\frac{ds}{dt} = -4a \frac{s^2 - v^2}{s^2 - D^2}. \quad (4)$$

Expressing u_y via s and D . Because $\vec{r}_1 - \vec{r}_2 = \vec{v}$ has no y component, the two slip velocities share the same y component, namely u_y . Geometrically, u_y is the altitude of the triangle with sides p, q and base v :

$$u_y = \frac{1}{2v} \sqrt{(s^2 - v^2)(v^2 - D^2)}. \quad (5)$$

Let $s_0 \equiv s(0) = u_0 + R$. As $t \rightarrow \infty$ the slip vanishes and $s \rightarrow v$.

Total displacement along y . The total displacement is

$$\ell_y = \int_0^\infty u_y dt = \int_{s=v}^{s_0} u_y \frac{dt}{ds} ds.$$

From (4), $dt/ds = -(s^2 - D^2)/[4a(s^2 - v^2)]$. Substituting (5) yields

$$\ell_y = \frac{\sqrt{v^2 - D^2}}{8av} \int_v^{s_0} \frac{s^2 - D^2}{\sqrt{s^2 - v^2}} ds.$$

With the change $s = v \cosh \theta$, one obtains the closed form

$$\ell_y = \frac{\sqrt{v^2 - D^2}}{8av} \left[\frac{s_0}{2} \sqrt{s_0^2 - v^2} + \left(\frac{v^2}{2} - D^2 \right) \operatorname{arcosh} \left(\frac{s_0}{v} \right) \right]. \quad (6)$$

Eliminating a using the given a_0 . The initial acceleration magnitude is

$$a_0 = \left| \ddot{\mathbf{u}}(0) \right| = a \left| \hat{r}_{1,0} + \hat{r}_{2,0} \right| = 2a \cos \frac{\varphi_0}{2}, \quad \cos \varphi_0 = \frac{u_0}{R}.$$

Hence $\cos(\varphi_0/2) = \sqrt{(1 + \cos \varphi_0)/2} = \sqrt{\frac{s_0}{2R}}$ and

$$\frac{1}{a} = \frac{\sqrt{2s_0/R}}{a_0}.$$

Also $D = u_0 - R$ and $\sqrt{v^2 - D^2} = \sqrt{2u_0(R - u_0)}$. Substituting these into (6), an equivalent and compact expression is

$$\ell_y = \frac{1}{4a_0} \sqrt{\frac{u_0}{R}} \left[\frac{s_0}{2} \sqrt{s_0^2 - v^2} + \left(\frac{v^2}{2} - (R - u_0)^2 \right) \operatorname{arcosh} \left(\frac{s_0}{v} \right) \right], \quad (7)$$

where $R = \sqrt{u_0^2 + v^2}$ and $s_0 = u_0 + R$.

Numerical evaluation. For $v = u_0 = 1 \text{ m/s}$ and $a_0 = 1 \text{ m/s}^2$:

$$R = \sqrt{2}, \quad s_0 = 1 + \sqrt{2} \quad \Rightarrow \quad \ell_y \approx \boxed{0.6631 \text{ m}}.$$