Physics Cup – TalTech 2019 – Problem 2. February 3, 2019

Typically, the Debye model describes well the heat capacity C_V of crystals: for absolute temperatures T much smaller than the so-called Debye temperature T_D, $C_V \propto T^3$; at the opposite limit of $T \gg T_D$, all the crystal lattice oscillations are thermally unlocked and hence, $C_V \approx 3Nk_B$, where N is the total number of atoms.

Surprisingly, the heat capacity of ice behaves in a totally different manner: with a very good precision and over a wide range of temperatures (from the melting point T_0 down to ca 100 K), its specific heat c_V is proportional to the absolute temperature. In what follows, you may assume that for ice, $c_V = \alpha T$, where $\alpha \approx 7.51\,\mathrm{J{\cdot}kg^{-1}K^{-2}}$, and the latent heat of melting $\lambda = 334\,\mathrm{kJ/kg}$.

Consider an isolated system consisting of equal masses m of water at temperature $T_0 = 273.15\,\mathrm{K}$, and ice. The ice is at a slightly lower temperature $T_0 - t$, where t is of the order of few kelvins. This isolated system includes also ideal reversible heat engines of negligible heat capacity. One heat engine is used to produce mechanical energy which is consumed by another heat engine which is operated as a refrigerator. What is the lowest temperature T which can be given to a n-th part of the ice (of mass m/n) if $n \gg 1$?

Hint 1 (20th Jan. 2019). Make use of the fact that apart from the conservation law of energy, the total entropy of the system will be either conserved or increased.

Hint 2 (27th Jan. 2019). Notice that at the final state, except for the small part of the ice which is cooled down, all the remaining water and ice should be brought to the same temperature T'. Indeed, otherwise the temperature differences could be used to operate a reversed heat engine as a fridge to further lower the temperature of the cooled ice.

Hint 3 (3rd Feb. 2019). Let us elaborate on the previous hint. Notice that $T' = T_0$. Indeed, it is easy to see that since t was supposed to be only of the order of few Kelvins, freezing all the water will release more heat than what is needed for bringing the temperature of ice up to the melting point, hence it is impossible to freeze all the water (cooling down a small portion of ice will release some further heat). So, at the final state, water and ice must co-exist, hence $T' = T_0$. Now, there are two unknown variables, the mass of water Δm which becomes frozen during the process (so that the final total mass of ice is $m + \Delta m$), and the temperature T. Hint 1 gives you all what is needed for finding Δm and T.

By the end of the third week of the second problem, there were 377 registered participants from 54 countries; among them there were 180 high school students, and 197 university students. During the three weeks, in total 31 solutions of the second problem were submitted, out of which 23 were correct. For the university students, there is still a chance of getting the speed bonus!

Correct solutions submitted by February 3, 2019:

Name	Uni/PreUni	country	subm. time (GMT)
Siddharth Tiwary	IIT Bombay	India	13 Jan. 2019 14:32
Vladislav Polyakov	PreUni	Russia	13 Jan. 2019 14:50
Thomas Foster	Oxford	UK	13 Jan. 2019 15:10
Oliver Lindström	PreUni	Sweden	13 Jan. 2019 15:54
Stefan Dolteanu	PreUni	Romania	13 Jan. 2019 16:14
Ivan Ridkokasha	Шевченко	Ukraine	13 Jan. 2019 16:55
Domagoj Perković	PreUni	Croatia	13 Jan. 2019 18:40
Johanes Suhardjo	HKUST	Indonesia	13 Jan. 2019 18:57
Gabriel Trigo	PreUni	Brazil	13 Jan. 2019 20:39
Ivander JM Waskito	PreUni	Indonesia	14 Jan. 2019 3:17
Eduard Burlacu	PreUni	Romania	14 Jan. 2019 8:49
Samarth Hawaldar	IISc	India	15 Jan. 2019 4:16
Bartłomiej Sikorski	PreUni	Poland	15 Jan. 2019 21:20
Oliwier Urbański	PreUni	Poland	18 Jan. 2019 11:56
Zăhărăchescu Mihai	PreUni	Romania	19 Jan. 2019 9:44
Felix Christensen	Oxford	UK	19 Jan. 2019 22:38
Petra Brčić	Univ. Zagreb	Croatia	20 Jan. 2019 18:59
Tùng Trần Xuân	PreUni	Vietnam	21 Jan. 2019 2:54
Muhamed Sokolović	PreUni	Bosnia & Herzeg.	21 Jan. 2019 19:31
Mateusz Kapusta	PreUni	Poland	22 Jan. 2019 20:26
Stephen Catsamas	PreUni	Australia	27 Jan. 2019 13:21
Gusti Atmaja	PreUni	Indonesia	27 Jan. 2019 15:58
Morteza Mudrick	PreUni	Indonesia	30 Jan. 2019 14:08