## Physics Cup - TalTech 2019 - Problem 5. May 5, 2019

Consider a check-board-like reflective diffraction grating a cross-section of which is shown in the figure below. The height of the reflecting surface above a reference plane is given by

$$
z=20 \lambda_{0}\left\lfloor\frac{x}{20 \lambda_{0}}\right\rfloor+\lambda_{0}\left\lfloor\frac{y}{20 \lambda_{0}}\right\rfloor,
$$

where $\lambda_{0}=500 \mathrm{~nm},\lfloor a\rfloor$ denotes the floor function (real number $a$ is rounded down to the nearest integer), and $|x|,|y|<5000 \lambda_{0}$ (thus, the size of the grating is $5 \mathrm{~mm} \times 5 \mathrm{~mm}$ ). The side surfaces (vertical edges in the figure below) of the "stairs" are black and absorb all the incident light. A parallel beam of white light containing all the wavelengths from $\lambda_{1}=400 \mathrm{~nm}$ to $\lambda_{2}=700 \mathrm{~nm}$ propagates parallel to the $z$-axis and falls onto the grating. The reflected beam is focused with a lens onto a screen perpendicular to the beam at $z=50 \mathrm{~cm}$. Sketch qualitatively the pattern which can be seen on the screen and show the approximate dimensions of the pattern. For each wavelength, mark only the main diffraction maximum (the brightest spot). Your answer should be a set of points at $x-y$ plane (at $z=50 \mathrm{~cm}$ ) showing the brightly illuminated region on the screen.

NB! The score for this problem is split into two parts: a sketch which captures the most important features of what can be on the screen will give a score of 0.5 , and a fully correct sketch - the remaining 0.5 pts . The speed bonus and the penalties for submitting wrong solutions will be applied separately to the both sub-scores.

The hint of 28th April First about how your answer should look like. For each wavelength, the brightest spot is basically a dot on the screen. As wavelength changes, the dot on the screen moves. So, the answer should be a line/curve or a set of curves. Second, in order to have a constructive interference, there should be a constructive interference for the beams adding up in the $x$-direction and in the $y$-direction (since any destructive superposition will negate anything there was before). Third, when figuring
out which of the spots is the brightest one, it might be helpful to recall the intensity distribution over angles for a diffraction grating.
By the end of the second week of the fifth problem, there were 405 registered participants from 55 countries; among them there were 204 high school students, and 201 university students. During the first two weeks, in total 15 solutions of the fifth problem were submitted, out of which 4 were correct.

Correct solutions submitted by 28th April 2019:
Solutions capturing the most important features of the pattern which can be seen on the screen (but possibly incorrect in details).

| Name | country | Uni/PreUni | subm. date/time (GMT) |
| :--- | :--- | :--- | :--- |
| Thomas Foster | UK | Oxford | 14 Apr. 2019 17:22 |
| Johanes Suhardjo | Indonesia | HKUST | 14 Apr. 2019 18:34 |
| Oliver Lindström | Sweden | PreUni | 16 Apr. 2019 10:19 |
| Oliwier Urbański | Poland | PreUni | 17 Apr. 2019 10:35 |
| Fully correct solutions. |  |  |  |
| Name | country | Uni/PreUni | subm. date/time (GMT) |
| Oliver Lindström | Sweden | PreUni | 16 Apr. 2019 10:19 |
| Johanes Suhardjo | Indonesia | HKUST | 20 Apr. 2019 14:27 |
| Oliwier Urbański | Poland | PreUni | 23 Apr. 2019 13:13 |
| Thomas Foster | UK | Oxford | 24 Apr. 2019 19:54 |

