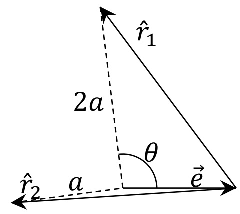
The eccentricity vector $\vec{e}$ is



$$\vec{e} = \frac{\vec{v} \times \vec{L}}{GMm} - \hat{r}$$

Because $\vec{v}_1$ and $\vec{v}_2$ are orthogonal and $|\vec{v}_1| = 2|\vec{v}_2|$, therefore $\vec{e} + \hat{r}_1$ and $\vec{e} + \hat{r}_2$ are orthogonal and $|\vec{e} + \hat{r}_1| = 2|\vec{e} + \hat{r}_2|$. Therefore

$$\begin{cases} e^2 + 4a^2 - 4ae\cos\theta = 1 \\ e^2 + a^2 + 2ae\sin\theta = 1 \end{cases}$$

$$\left(\frac{e^2 + 4a^2 - 1}{4ae}\right)^2 + \left(\frac{e^2 + a^2 - 1}{2ae}\right)^2 = 1$$

$$20a^4 - 16a^2 + 5(e^2 - 1)^2 = 0$$

$$\Delta = 256 - 400(e^2 - 1)^2 \geq 0$$

$$\frac{\sqrt{5}}{5} \leq e \leq \frac{3\sqrt{5}}{5}$$

Therefore, the smallest possible eccentricity of the orbit is $\frac{\sqrt{5}}{5}$.