Problem NO 3 Wang Huiyuan

For 8 points in a plane with 8 constraints (rods in between)
there are $2 \times 8 - 8 = 8$ degrees of freedom

8 degrees of freedom $\Rightarrow$ 8 characteristic frequencies

Out of which there should be 3 trivial frequency with 0 rad s^{-1}
they are: x-translation, y-translation, rotation in the plane.

Hence, there are exactly 5 non-trivial vibration frequencies.

Use Lagrangian method to solve for normal modes,

let KE be $T = \frac{1}{2} \dot{q}^T M \dot{q}$

PE be $U = U_0 + \underbrace{\sum_i q_i \frac{\partial U}{\partial q_i}\Big|_{eq}}_{0 \text{ at eq}} + \frac{1}{2} \sum_{ij} q_i q_j \frac{\partial^2 U}{\partial q_i \partial q_j}\Big|_{eq}$

let $K_{ij} = \frac{\partial^2 U}{\partial q_i \partial q_j}\Big|_{eq}$

$$\therefore \frac{dE}{dt} = \sum_{ij} \left(M_{ij} \ddot{q}_j + K_{ij} q_j \right) \dot{q}_i = 0$$

$$\therefore M \ddot{q} + K q = 0$$

$$\therefore \omega^2 q - M^{-1} K q = 0$$

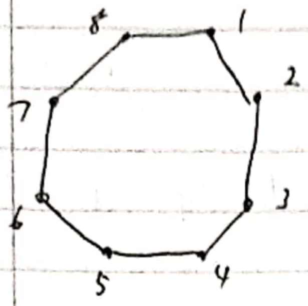
$$(\omega^2 I - M^{-1} K) q = 0$$

$$|\omega^2 I - M^{-1} K| = 0$$

solve this determinant would give ω^2

Find K matrix and M matrix in this problem,

First set up the problem as eight points (x_i, y_i) $i \in [1, 8]$



Because the rigid rod makes coordinate difficult to choose, we first generalize the problem by allowing points connected by spring with constant k_1, then return to rod by $k_1 \to +\infty$ would be equivalent to a perfect rigid rod.

So there are now 16 dof in the general problem

KE = Translational KE + Rotational KE

$$\text{Translational KE} = \sum_{i=1}^{8} \frac{1}{2} m \left[\left(\frac{V_{x_i} + V_{x_{i+1}}}{2} \right)^2 + \left(\frac{V_{y_i} + V_{y_{i+1}}}{2} \right)^2 \right]$$

index cyclic here

Since V_{CoM} of rod is half the speed of two ends summed

$$\text{Rotational KE} = \sum_{i=1}^{8} \frac{1}{2} I \left[\left(\frac{V_{x_i} - V_{x_{i+1}}}{l} \right)^2 + \left(\frac{V_{y_i} - V_{y_{i+1}}}{l} \right)^2 \right]$$

since $|\vec{V}_i - \vec{V}_{i+1}| = |\omega l|$

Also, $I = \frac{1}{12} m l^2$

$$M_{ij} = \frac{\partial^2 KE}{\partial \dot{q}_i \partial \dot{q}_j}$$

Note that $\{\dot{q}_i\} = \{\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2, \dots, \dot{x}_8, \dot{y}_8\}$

16 variables

PE = Spring PE + Torsional PE

$$\text{Spring PE} = \sum_{i=1}^{8} \frac{1}{2} K_1 \left[\sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} - l\right]^2$$

(index cyclic)

$$\text{Torsional PE} = \sum_{i=1}^{8} \frac{1}{2} K_2 \left(\theta_i - \frac{3}{4}\pi\right)^2$$

$$= \sum_{i=1}^{8} \frac{1}{2} K_2 \left[\arccos\left[\frac{(\vec{r}_{i+1} - \vec{r}_i)\cdot(\vec{r}_{i-1} - \vec{r}_i)}{|\vec{r}_{i+1} - \vec{r}_i||\vec{r}_{i-1} - \vec{r}_i|}\right] - \frac{3}{4}\pi\right]^2$$

$$K_{ij} = \frac{\partial^2 PE}{\partial q_i \partial q_j}$$

Then solve on mathematica $|w^2 I - M^{-1}K| = 0$

and $\lim_{k_1 \to \infty}$ eigenvalues

In[ ]:=
```
TKE = Sum[1 / 2 * m * (Subscript[vx, i] + Subscript[vx, i + 1]) ^ 2 / 4 + 1 / 2 * m * (Subscript[vy, i] + Subscript[vy, i + 1]) ^ 2 / 4, {i, 1, 7}] +
  1 / 2 * m * (Subscript[vx, 8] + Subscript[vx, 1]) ^ 2 / 4 + 1 / 2 * m * (Subscript[vy, 8] + Subscript[vy, 1]) ^ 2 / 4
```

Out[ ]=

$$\frac{1}{8} m (vx_1 + vx_2)^2 + \frac{1}{8} m (vx_2 + vx_3)^2 + \frac{1}{8} m (vx_3 + vx_4)^2 + \frac{1}{8} m (vx_4 + vx_5)^2 + \frac{1}{8} m (vx_5 + vx_6)^2 + \frac{1}{8} m (vx_6 + vx_7)^2 + \frac{1}{8} m (vx_1 + vx_8)^2 + \frac{1}{8} m (vx_7 + vx_8)^2 + \frac{1}{8} m (vy_1 + vy_2)^2 + \frac{1}{8} m (vy_2 + vy_3)^2 + \frac{1}{8} m (vy_3 + vy_4)^2 + \frac{1}{8} m (vy_4 + vy_5)^2 + \frac{1}{8} m (vy_5 + vy_6)^2 + \frac{1}{8} m (vy_6 + vy_7)^2 + \frac{1}{8} m (vy_1 + vy_8)^2 + \frac{1}{8} m (vy_7 + vy_8)^2$$

In[ ]:=
```
RKE = Sum[1 / 24 * m * (Subscript[vx, i] - Subscript[vx, i + 1]) ^ 2 + 1 / 24 * m * (Subscript[vy, i] - Subscript[vy, i + 1]) ^ 2, {i, 1, 7}] +
  1 / 24 * m * (Subscript[vx, 8] - Subscript[vx, 1]) ^ 2 + 1 / 24 * m * (Subscript[vy, 8] - Subscript[vy, 1]) ^ 2
```

Out[ ]=

$$\frac{1}{24} m (vx_1 - vx_2)^2 + \frac{1}{24} m (vx_2 - vx_3)^2 + \frac{1}{24} m (vx_3 - vx_4)^2 + \frac{1}{24} m (vx_4 - vx_5)^2 + \frac{1}{24} m (vx_5 - vx_6)^2 + \frac{1}{24} m (vx_6 - vx_7)^2 + \frac{1}{24} m (vx_7 - vx_8)^2 + \frac{1}{24} m (-vx_1 + vx_8)^2 + \frac{1}{24} m (vy_1 - vy_2)^2 + \frac{1}{24} m (vy_2 - vy_3)^2 + \frac{1}{24} m (vy_3 - vy_4)^2 + \frac{1}{24} m (vy_4 - vy_5)^2 + \frac{1}{24} m (vy_5 - vy_6)^2 + \frac{1}{24} m (vy_6 - vy_7)^2 + \frac{1}{24} m (vy_7 - vy_8)^2 + \frac{1}{24} m (-vy_1 + vy_8)^2$$

In[ ]:=
```
KE = TKE + RKE
```

Out[ ]=

$$\frac{1}{24} m (vx_1 - vx_2)^2 + \frac{1}{8} m (vx_1 + vx_2)^2 + \frac{1}{24} m (vx_2 - vx_3)^2 + \frac{1}{8} m (vx_2 + vx_3)^2 + \frac{1}{24} m (vx_3 - vx_4)^2 + \frac{1}{8} m (vx_3 + vx_4)^2 + \frac{1}{24} m (vx_4 - vx_5)^2 + \frac{1}{8} m (vx_4 + vx_5)^2 + \frac{1}{24} m (vx_5 - vx_6)^2 + \frac{1}{8} m (vx_5 + vx_6)^2 + \frac{1}{24} m (vx_6 - vx_7)^2 + \frac{1}{8} m (vx_6 + vx_7)^2 + \frac{1}{24} m (vx_7 - vx_8)^2 + \frac{1}{24} m (-vx_1 + vx_8)^2 + \frac{1}{8} m (vx_1 + vx_8)^2 + \frac{1}{8} m (vx_7 + vx_8)^2 + \frac{1}{24} m (vy_1 - vy_2)^2 + \frac{1}{8} m (vy_1 + vy_2)^2 + \frac{1}{24} m (vy_2 - vy_3)^2 + \frac{1}{8} m (vy_2 + vy_3)^2 + \frac{1}{24} m (vy_3 - vy_4)^2 + \frac{1}{8} m (vy_3 + vy_4)^2 + \frac{1}{24} m (vy_4 - vy_5)^2 + \frac{1}{8} m (vy_4 + vy_5)^2 + \frac{1}{24} m (vy_5 - vy_6)^2 + \frac{1}{8} m (vy_5 + vy_6)^2 + \frac{1}{24} m (vy_6 - vy_7)^2 + \frac{1}{8} m (vy_6 + vy_7)^2 + \frac{1}{24} m (vy_7 - vy_8)^2 + \frac{1}{24} m (-vy_1 + vy_8)^2 + \frac{1}{8} m (vy_1 + vy_8)^2 + \frac{1}{8} m (vy_7 + vy_8)^2$$

In[ ]:=
```
TPE = Sum[1/2*Subscript[k, 1]*(Sqrt[(Subscript[x, i] - Subscript[x, i+1])^2 + (Subscript[y, i] - Subscript[y, i+1])^2] - l)^2, {i, 1, 7}] +
  1/2*Subscript[k, 1]*(Sqrt[(Subscript[x, 8] - Subscript[x, 1])^2 + (Subscript[y, 8] - Subscript[y, 1])^2] - l)^2
```

Out[ ]=

$$\frac{1}{2}k_1\left(-l+\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right)^2+\frac{1}{2}k_1\left(-l+\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}\right)^2+\frac{1}{2}k_1\left(-l+\sqrt{(x_3-x_4)^2+(y_3-y_4)^2}\right)^2+\frac{1}{2}k_1\left(-l+\sqrt{(x_4-x_5)^2+(y_4-y_5)^2}\right)^2+$$
$$\frac{1}{2}k_1\left(-l+\sqrt{(x_5-x_6)^2+(y_5-y_6)^2}\right)^2+\frac{1}{2}k_1\left(-l+\sqrt{(x_6-x_7)^2+(y_6-y_7)^2}\right)^2+\frac{1}{2}k_1\left(-l+\sqrt{(x_7-x_8)^2+(y_7-y_8)^2}\right)^2+\frac{1}{2}k_1\left(-l+\sqrt{(-x_1+x_8)^2+(-y_1+y_8)^2}\right)^2$$

In[ ]:=
```
Angle[i_] :=
  ArcCos[(((Subscript[x, i+1] - Subscript[x, i])*(Subscript[x, i-1] - Subscript[x, i]) + (Subscript[y, i+1] - Subscript[y, i])*(Subscript[y, i-1] - Subscript[y, i]))/
    (Sqrt[(Subscript[x, i+1] - Subscript[x, i])^2 + (Subscript[y, i+1] - Subscript[y, i])^2]*
      Sqrt[(Subscript[x, i-1] - Subscript[x, i])^2 + (Subscript[y, i-1] - Subscript[y, i])^2]))]
```

In[ ]:=
```
RPE = Sum[1/2*Subscript[k, 2]*(Angle[i] - 3*π/4)^2, {i, 2, 7}] +
  1/2*Subscript[k, 2]*
   (ArcCos[(((Subscript[x, 1] - Subscript[x, 8])*(Subscript[x, 8-1] - Subscript[x, 8]) + (Subscript[y, 1] - Subscript[y, 8])*(Subscript[y, 8-1] - Subscript[y, 8]))/
       (Sqrt[(Subscript[x, 1] - Subscript[x, 8])^2 + (Subscript[y, 1] - Subscript[y, 8])^2]*
         Sqrt[(Subscript[x, 8-1] - Subscript[x, 8])^2 + (Subscript[y, 8-1] - Subscript[y, 8])^2]))] - 3*π/4)^2 +
  1/2*Subscript[k, 2]*
   (ArcCos[(((Subscript[x, 2] - Subscript[x, 1])*(Subscript[x, 8] - Subscript[x, 1]) + (Subscript[y, 2] - Subscript[y, 1])*(Subscript[y, 8] - Subscript[y, 1]))/
       (Sqrt[(Subscript[x, 2] - Subscript[x, 1])^2 + (Subscript[y, 2] - Subscript[y, 1])^2]*
         Sqrt[(Subscript[x, 8] - Subscript[x, 1])^2 + (Subscript[y, 8] - Subscript[y, 1])^2]))] - 3*π/4)^2
```

Out[ ]=

$$\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_1-x_2)(-x_2+x_3)+(y_1-y_2)(-y_2+y_3)}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\sqrt{(-x_2+x_3)^2+(-y_2+y_3)^2}}\right]\right)^2k_2+\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_2-x_3)(-x_3+x_4)+(y_2-y_3)(-y_3+y_4)}{\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}\sqrt{(-x_3+x_4)^2+(-y_3+y_4)^2}}\right]\right)^2k_2+$$
$$\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_3-x_4)(-x_4+x_5)+(y_3-y_4)(-y_4+y_5)}{\sqrt{(x_3-x_4)^2+(y_3-y_4)^2}\sqrt{(-x_4+x_5)^2+(-y_4+y_5)^2}}\right]\right)^2k_2+\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_4-x_5)(-x_5+x_6)+(y_4-y_5)(-y_5+y_6)}{\sqrt{(x_4-x_5)^2+(y_4-y_5)^2}\sqrt{(-x_5+x_6)^2+(-y_5+y_6)^2}}\right]\right)^2k_2+$$
$$\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_5-x_6)(-x_6+x_7)+(y_5-y_6)(-y_6+y_7)}{\sqrt{(x_5-x_6)^2+(y_5-y_6)^2}\sqrt{(-x_6+x_7)^2+(-y_6+y_7)^2}}\right]\right)^2k_2+\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_1-x_8)(x_7-x_8)+(y_1-y_8)(y_7-y_8)}{\sqrt{(x_1-x_8)^2+(y_1-y_8)^2}\sqrt{(x_7-x_8)^2+(y_7-y_8)^2}}\right]\right)^2k_2+$$
$$\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(-x_1+x_2)(-x_1+x_8)+(-y_1+y_2)(-y_1+y_8)}{\sqrt{(-x_1+x_2)^2+(-y_1+y_2)^2}\sqrt{(-x_1+x_8)^2+(-y_1+y_8)^2}}\right]\right)^2k_2+\frac{1}{2}\left(-\frac{3\pi}{4}+\text{ArcCos}\left[\frac{(x_6-x_7)(-x_7+x_8)+(y_6-y_7)(-y_7+y_8)}{\sqrt{(x_6-x_7)^2+(y_6-y_7)^2}\sqrt{(-x_7+x_8)^2+(-y_7+y_8)^2}}\right]\right)^2k_2$$

In[ ]:=
```
PE = TPE + RPE
```

```
In[•]:= Mmat =
  D[KE, {{Subscript[vx, 1], Subscript[vy, 1], Subscript[vx, 2], Subscript[vy, 2], Subscript[vx, 3], Subscript[vy, 3], Subscript[vx, 4], Subscript[vy, 4],
     Subscript[vx, 5], Subscript[vy, 5], Subscript[vx, 6], Subscript[vy, 6], Subscript[vx, 7], Subscript[vy, 7], Subscript[vx, 8], Subscript[vy, 8]}, 2}]
```

```
Out[•]= {{2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, m/6, 0}, {0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, m/6},
  {m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0, m/6}, {m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3, 0}, {0, m/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, m/6, 0, 2m/3}}
```

```
In[•]:= Kmat =
  D[PE, {{Subscript[x, 1], Subscript[y, 1], Subscript[x, 2], Subscript[y, 2], Subscript[x, 3], Subscript[y, 3], Subscript[x, 4], Subscript[y, 4],
     Subscript[x, 5], Subscript[y, 5], Subscript[x, 6], Subscript[y, 6], Subscript[x, 7], Subscript[y, 7], Subscript[x, 8], Subscript[y, 8]}, 2}]
```

```
Out[•]= {<< 1 >>}
  large output    show less    show more    show all    set size limit...
```

```
In[19]:= Subscript[x, 1] = l/2; Subscript[y, 1] = l/2*(1+Sqrt[2]);
  Subscript[x, 8] = -l/2; Subscript[y, 8] = l/2*(1+Sqrt[2]);
  Subscript[x, 4] = l/2; Subscript[y, 4] = -l/2*(1+Sqrt[2]);
  Subscript[x, 5] = -l/2; Subscript[y, 5] = -l/2*(1+Sqrt[2]);
  Subscript[y, 2] = l/2; Subscript[x, 2] = l/2*(1+Sqrt[2]);
  Subscript[y, 3] = -l/2; Subscript[x, 3] = l/2*(1+Sqrt[2]);
  Subscript[y, 7] = l/2; Subscript[x, 7] = -l/2*(1+Sqrt[2]);
  Subscript[y, 6] = -l/2; Subscript[x, 6] = -l/2*(1+Sqrt[2]);
```

```
In[•]:= Kmat = FullSimplify[Kmat, l > 0]
```

In[ ]:=
```
IMK = Dot[Inverse[Mmat], Kmat]
```

Out[ ]=

$$\left\{\left\{\frac{13 k_1}{28 m}+\frac{k_2}{8\sqrt{2}\, l^2 m}+\frac{97\left(\frac{3 k_1}{2}+\frac{k_2}{l^2}\right)}{56 m}-\frac{13\left(-\frac{k_1}{2}-\frac{(2+\sqrt{2})\, k_2}{2 l^2}\right)}{28 m},\ \cdots 14 \cdots,\ -\frac{121 k_2}{56\sqrt{2}\, l^2 m}+\frac{-\frac{k_1}{2}+\frac{(1+\sqrt{2})\, k_2}{l^2}}{8 m}-\frac{13\left(k_1-\frac{(2+\sqrt{2})\, k_2}{l^2}\right)}{56 m}\right\},\right.$$

$$\left.\cdots 14 \cdots,\ \left\{-\frac{97 k_2}{56\sqrt{2}\, l^2 m}+\frac{\frac{k_1}{2}-\frac{k_2}{l^2}}{8 m}-\frac{13\left(-k_1+\frac{\cdots 1 \cdots}{l^2}\right)}{56 m},\ \cdots 14 \cdots,\ \frac{k_2}{8\sqrt{2}\, l^2 m}+\frac{13(2+\sqrt{2})\, k_2}{28\, l^2 m}+\frac{13 \cdots 1 \cdots}{\cdots 1 \cdots}+\frac{97\left(\frac{\cdots 1 \cdots}{2}+\cdots 1 \cdots\right)}{56 m}\right\}\right\}$$

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In[2]:=
```
EigenSet = Eigenvalues[IMK]
```

Out[2]=
```
Eigenvalues[IMK]
```

$$\text{EigenSet} = \left\{0,\ 0,\ 0,\ -\frac{6\left(-5\,l^2\,k_1 + 3\,\sqrt{2}\,l^2\,k_1\right)}{7\,l^2\,m},\ \frac{6\left(5\,l^2\,k_1 + 3\,\sqrt{2}\,l^2\,k_1\right)}{7\,l^2\,m},\ \frac{3\left(l^2\,k_1 + 2\,k_2 - \sqrt{2}\,k_2\right)}{2\,l^2\,m},\ \frac{3\left(l^2\,k_1 + 2\,k_2 - \sqrt{2}\,k_2\right)}{2\,l^2\,m},\ \frac{24\left(27\,k_2 + 19\,\sqrt{2}\,k_2\right)}{7\left(3 + 2\,\sqrt{2}\right)l^2\,m},\right.$$

$$\frac{3\left(5\,l^2\,k_1 + 10\,k_2 - \sqrt{25\,l^4\,k_1^2 + 44\,l^2\,k_1\,k_2 + 100\,k_2^2}\right)}{7\,l^2\,m},\ \frac{3\left(5\,l^2\,k_1 + 10\,k_2 - \sqrt{25\,l^4\,k_1^2 + 44\,l^2\,k_1\,k_2 + 100\,k_2^2}\right)}{7\,l^2\,m},\ \frac{3\left(5\,l^2\,k_1 + 10\,k_2 + \sqrt{25\,l^4\,k_1^2 + 44\,l^2\,k_1\,k_2 + 100\,k_2^2}\right)}{7\,l^2\,m},$$

$$\frac{3\left(5\,l^2\,k_1 + 10\,k_2 + \sqrt{25\,l^4\,k_1^2 + 44\,l^2\,k_1\,k_2 + 100\,k_2^2}\right)}{7\,l^2\,m},\ \frac{1}{224\left(-2 + \sqrt{2}\right)l^2\,m}3\left(-\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 - 560\,k_2 - \sqrt{401408\left(-2 + \sqrt{2}\right)l^2\,k_1\,k_2 + \left(\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 + 560\,k_2\right)^2}\right),$$

$$\frac{1}{224\left(-2 + \sqrt{2}\right)l^2\,m}3\left(-\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 - 560\,k_2 - \sqrt{401408\left(-2 + \sqrt{2}\right)l^2\,k_1\,k_2 + \left(\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 + 560\,k_2\right)^2}\right),\ \frac{1}{224\left(-2 + \sqrt{2}\right)l^2\,m}$$

$$3\left(-\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 - 560\,k_2 + \sqrt{401408\left(-2 + \sqrt{2}\right)l^2\,k_1\,k_2 + \left(\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 + 560\,k_2\right)^2}\right),\ \frac{1}{224\left(-2 + \sqrt{2}\right)l^2\,m}$$

$$\left.3\left(-\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 - 560\,k_2 + \sqrt{401408\left(-2 + \sqrt{2}\right)l^2\,k_1\,k_2 + \left(\left(560 - 280\,\sqrt{2}\right)l^2\,k_1 + 560\,k_2\right)^2}\right)\right\}$$

In[ ]:=
```
EigenSet = Limit[EigenSet, Subscript[k, 1] → +∞]
```

Out[1]=

$$\left\{0, 0, 0, \frac{\infty}{\text{Sign}[m]}, \frac{\infty}{\text{Sign}[m]}, \frac{\infty}{\text{Sign}[m]}, \frac{\infty}{\text{Sign}[m]}, \frac{24\left(5+3\sqrt{2}\right)k_2}{7\,l^2\,m}, \frac{12\,k_2}{5\,l^2\,m}, \frac{12\,k_2}{5\,l^2\,m}, \frac{\infty}{l^2\,m}, \frac{\infty}{l^2\,m}, \frac{\infty}{\text{Sign}[m]}, \frac{\infty}{\text{Sign}[m]}, \frac{48\,k_2}{l^2\left(10\,m-5\sqrt{2}\,m\right)}, \frac{48\,k_2}{l^2\left(10\,m-5\sqrt{2}\,m\right)}\right\}$$

There are five vibration modes (with three distinct frequencies)
(from $\lim_{K_{rod}\to\infty} \lambda_i$, finite means satisfy rigid rod, infinite solution eliminated).

$$w_1 = \sqrt{\frac{24(5+3\sqrt{2})K}{7ml^2}}$$ degeneracy: 1

$$w_2 = w_3 = \sqrt{\frac{12K}{5ml^2}}$$ degeneracy: 2

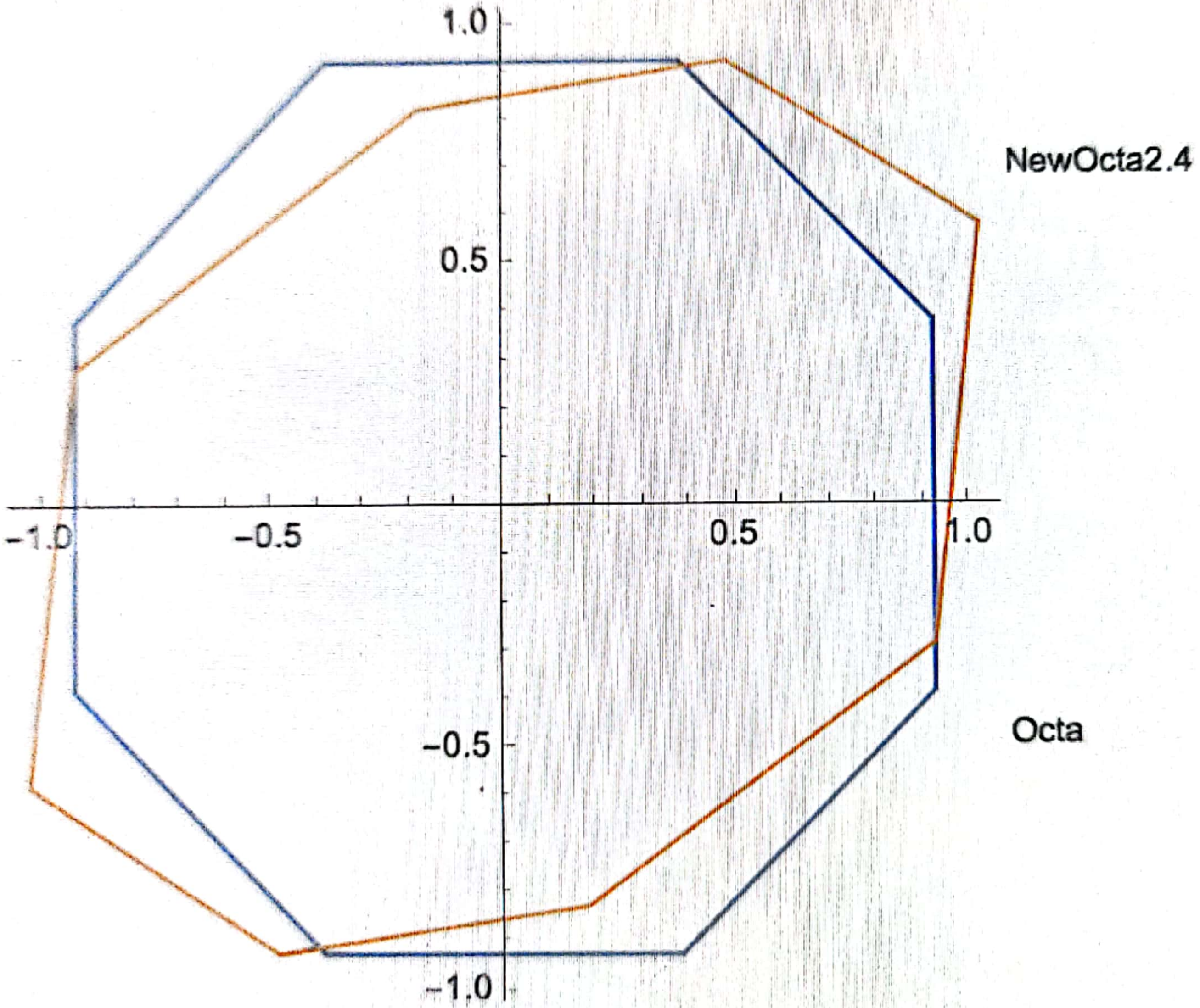
$$w_4 = w_5 = \sqrt{\frac{48K}{(10-5\sqrt{2})ml^2}}$$ degeneracy: 2

The demonstration of these 5 modes ~~are attached.~~
the two lowest frquency is basically stretching in one direction
the two intermediate frequency has more nodes and is like a triangle
the highest frequency has points moving in and out alternatingly

```
{-1.02388, -0.582676}, {-0.92388, 0.282683}, {-0.182691, 0.8
```

In[147]:=
```
ListLinePlot[{Labeled[Octa, "Octa"], Labeled[NewOcta, "NewOct
```

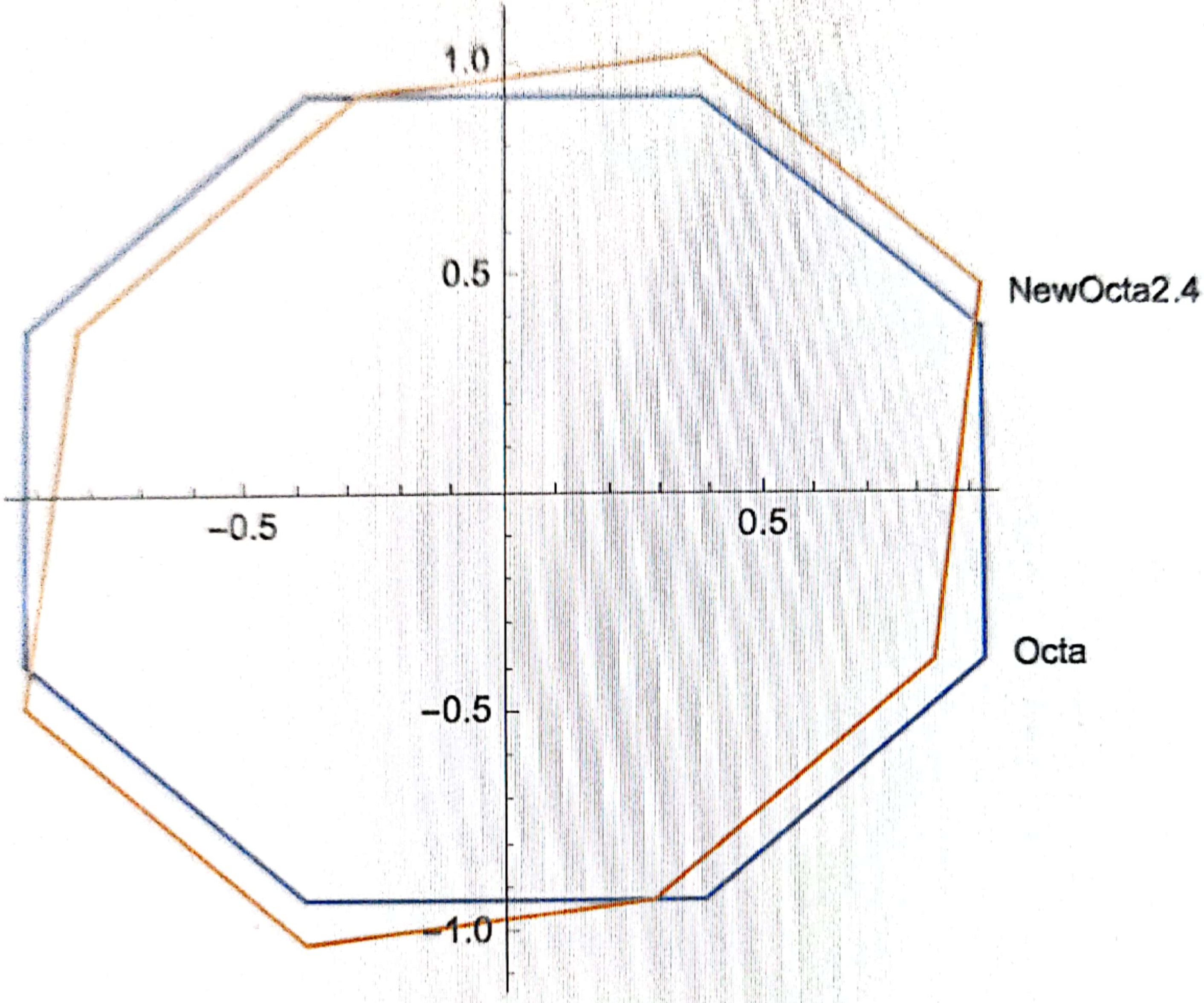
Out[147]=



In[148]:=
```
delta2 = Partition[{-0.000023999558413101626`, 0.9999760010l7
```

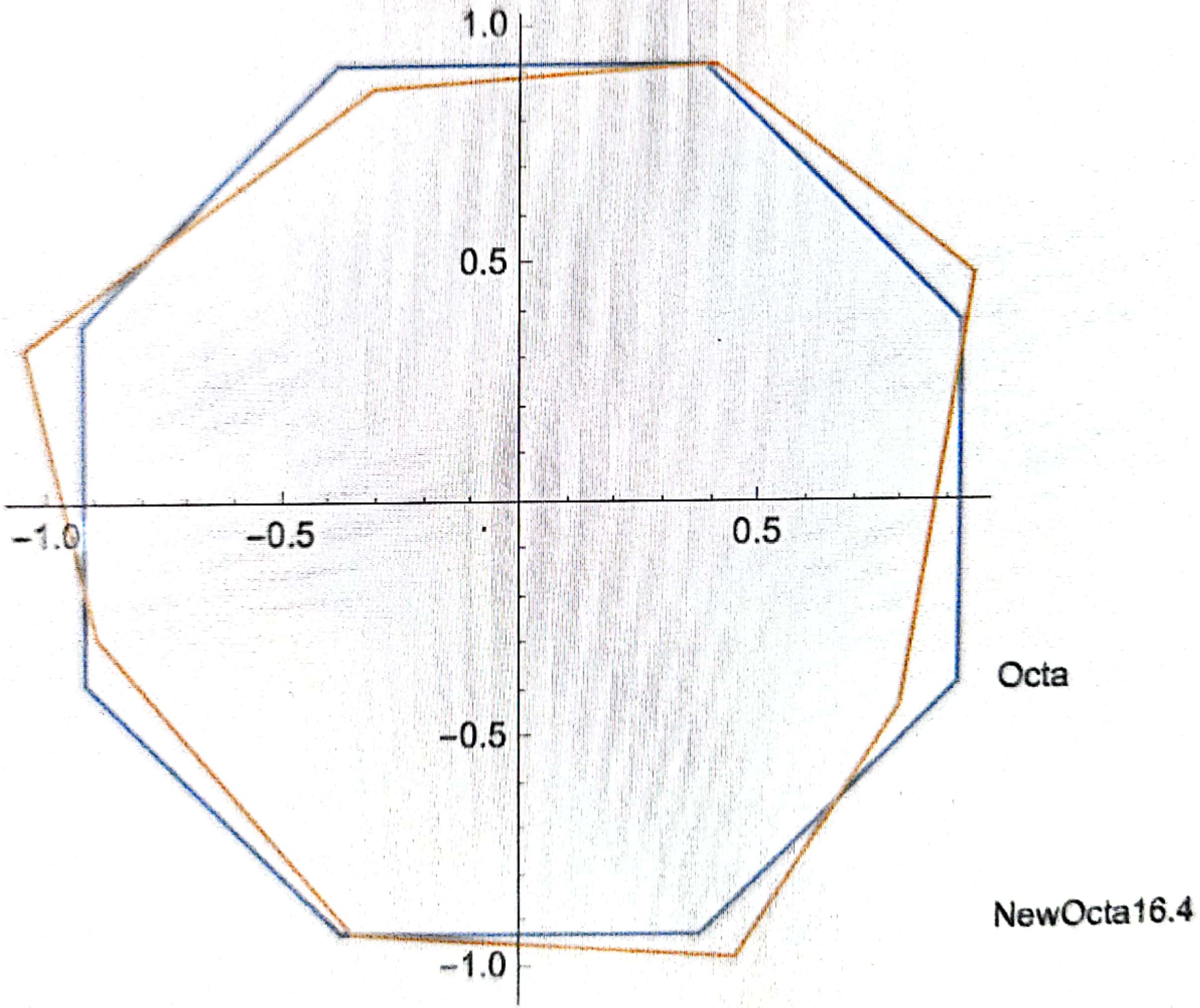
扫描全能王 创建

Out[150]=



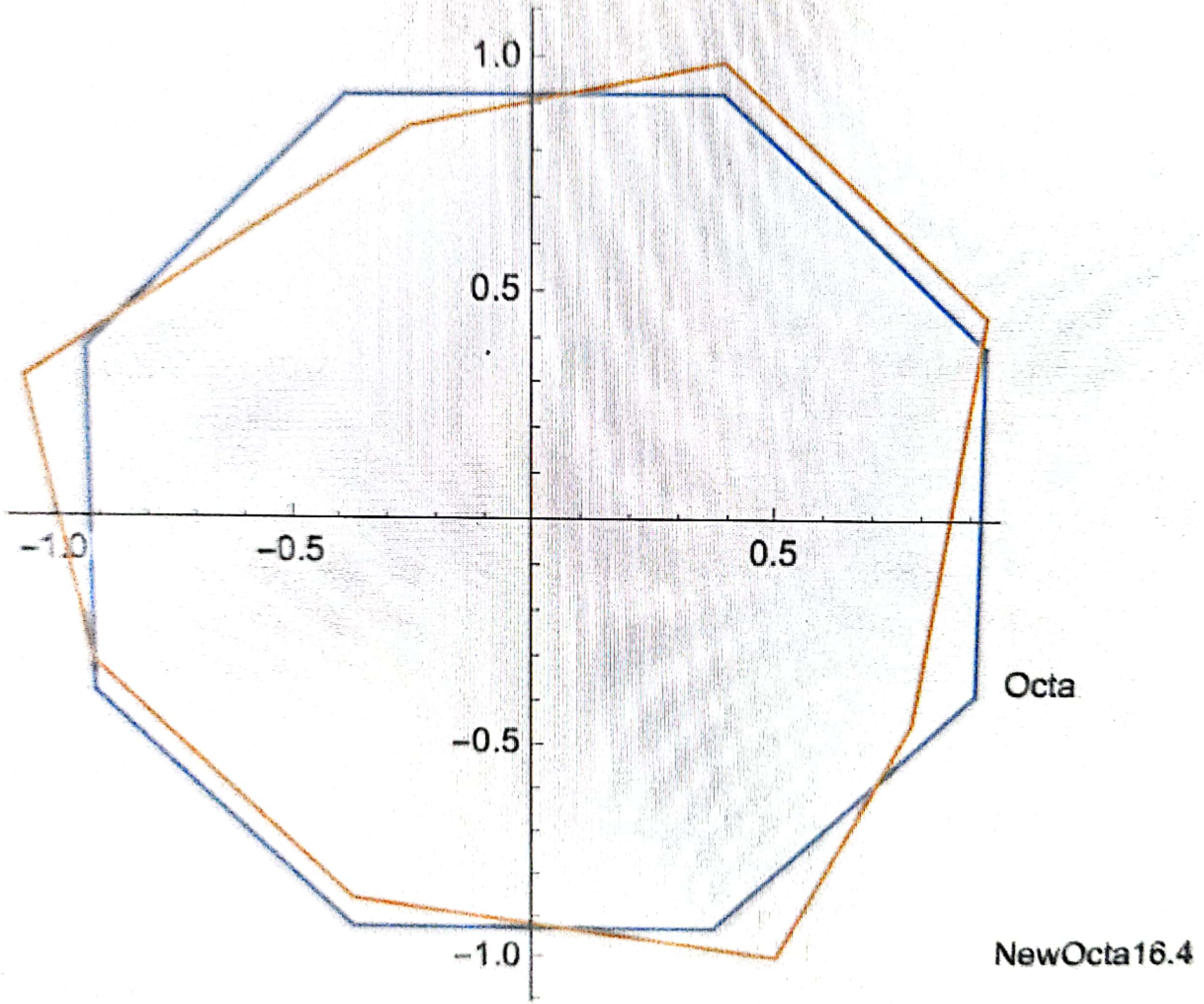
```
NewOcta3 = Octa + delta3 / 40;
ListLinePlot[{Labeled[Octa, "Octa"], Labeled[NewOcta3
```

Out[153]=



```
ListLinePlot[{Labeled[Octa, "Octa"], Labeled[NewOcta
```

Out[156]=




```
In[157]:= delta5 = Partition[
```

Out[159]=