We use $c=1$

We use frames O, A, B, C, and D where the spaceship is instantaneously at rest at 0, τ, 2τ, 3τ, and 4τ, respectively, and the 4-velocities of these frames $v_O^\alpha, v_A^\alpha, v_B^\alpha, v_C^\alpha, v_D^\alpha$
We use Einstein's convention for dot products in Minkowski space so that $u_\alpha w^\alpha$ gives the Lorentz 4-invariant. We omit the z-component.
Below is a table for the 4-velocity components in different frames:

FRAME:	O	A	B
v_O^α	$v_O^O=(1,0,0)$	$v_O^A\ (\gamma,-\gamma v,0)$	$v_O^B=(a,b,c)=(\gamma^2,-\gamma v,-\gamma^2 v)$
v_A^α	$v_A^O=(\gamma,\gamma v,0)$	$v_A^A=(1,0,0)$	$v_A^B=(\gamma,0,-\gamma v)$
v_B^α	$v_B^O=(\gamma^2,\gamma^2 v,\gamma v)$ by analogy with v_O^B	$v_B^A=(\gamma,0,\gamma v)$	$v_B^B=(1,0,0)$

4-invariants

$$v_O^{O\alpha} v^O_{A\alpha} = -\gamma = v_O^{A\alpha} v^B_{A\alpha} = -a\gamma - \gamma v c \Rightarrow 1 = a + vc$$

$$v_O^{A\alpha} v^A_{B\alpha} = -\gamma^2 = v_O^{B\alpha} v^B_{B\alpha} = -a \Rightarrow \underline{a=\gamma^2} \Rightarrow \underline{c} = \frac{1-\gamma^2}{v} = \frac{-v}{1-v^2} = \underline{-\gamma^2 v}$$

$$-1 = v_O^{B\alpha} v^B_{O\alpha} = -a^2+b^2+c^2 = b^2+\gamma^4 v^2-\gamma^4 = b^2-\gamma^2 \Rightarrow b^2=\gamma^2-1=v^2\gamma^2 \Rightarrow \underline{b=-\gamma v}$$

another table

FRAME:	O	B	D
v_O^α	$v_O^O=(1,0,0)$	$v_O^B=(\gamma^2,-\gamma v,-\gamma^2 v)$	$v_O^D=(\gamma,\ d,\ e)$
v_D^α	by analogy with v_B^O →	$v_D^B=(\gamma^2,-\gamma^2 v,-\gamma v)$	$v_D^D=(1,0,0)$

$$v_D^{B\alpha} v^B_{O\alpha} = -\gamma^4+2\gamma^3 v^2 = v_O^{D\alpha} v^D_{D\alpha} = -\gamma \Rightarrow \gamma = \gamma^3(\gamma-2v^2) \Rightarrow 1-v^2 = \gamma-2v^2 \Rightarrow$$

$$\Rightarrow \gamma = 1+v^2 \Rightarrow 1 = (1-v^2)(1+v^2)^2 = (1-v^4)(1+v^2) \overset{v^2=x}{=} (1-x^2)(1+x) = 1+x-x^2-x^3 \Rightarrow$$

$$\Rightarrow x^2+x-1=0 \underset{(x<1)}{\Rightarrow} x = \frac{\sqrt5-1}{2} \Rightarrow v = \sqrt{\frac{\sqrt5-1}{2}} \simeq 0.786$$