Physics Cup 2023
Problem 5: Light sources

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1 Introduction

We assume intensity doesn't depend on distance, as said in question. We know for any pair of point sources of light constructive interference occurs when the path difference between the light coming from two sources is an integral multiple of the wavelength. They form **hyperbolic fringes**. This is because the locus of all points whose difference in distance from two fixed points in a plane is a hyperbola. So for two sources, we should get a family of hyperbolas (for different wavelength multiples as the path difference).

Claim: For three sources, *each maxima point must lie on three hyperbolas.*
This is because light from all three sources must be in the same phase (i.e. with some $n\lambda$ path difference) for a point to be a local maxima of intensity. This proves that we will find the maxima point lying on a hyperbola for any choice of two focuses. Since there are three possible ways of choosing two sources as a focus and getting a family of hyperbolas with them as the focus (each representing some $2n\pi$ phase difference), we get three hyperbolas at each maximum.

And indeed, from the given chart of points, I found three distinct families of hyperbolas by inspection.

2 Finding three sources

I will describe how I found the focus points of the three hyperbola families, thus giving me the three sources.

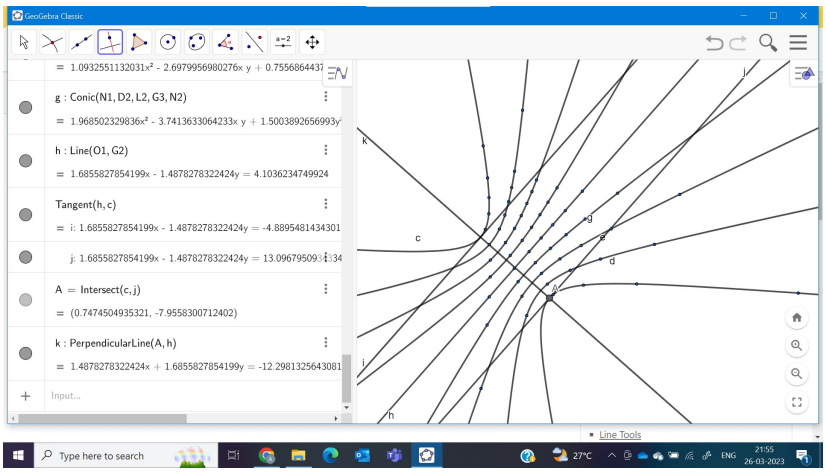

Figure 1: Finding the transverse axis of first hyperbola family

Very conveniently, each family of hyperbola had their conjugate axis among the set of points. The conjugate axis represents the special case where the difference in distance from two foci is zero. Now I constructed a tangent to one of the hyperbolas parallel to the conjugate axis. From the point where it touches the hyperbola, I constructed a line perpendicular to the conjugate axis; this is precisely the transverse axis containing the two foci (sources).

In this same process, I found the transverse axis of the other two families as well.

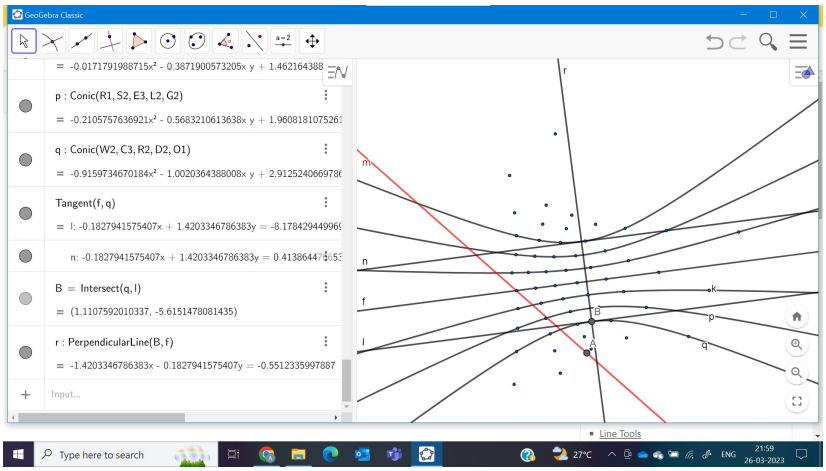

Figure 2: Finding the transverse axis of second hyperbola family

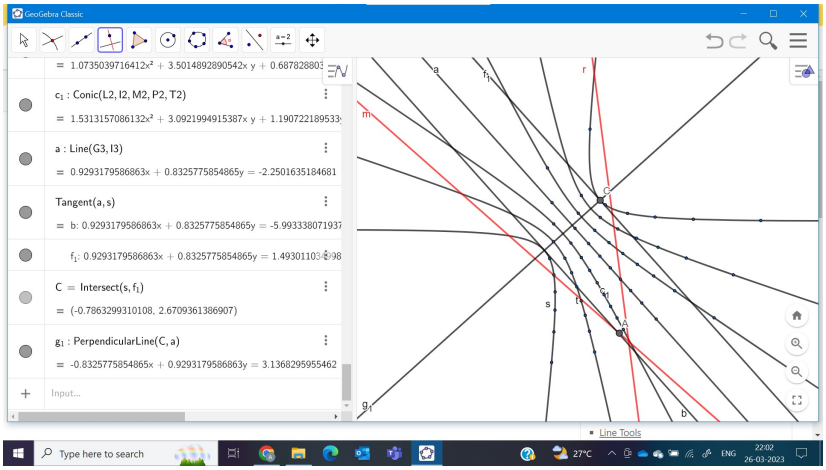

Figure 3: Finding the transverse axis of third hyperbola family

The focus points are the triangle's vertices formed by the three transverse axes (see figure 4, 5). They are labelled D, E, and F in figure 4. To verify if they are the correct foci, I chose any two at random (e.g. D and E) and plotted a hyperbola using them as foci and ***any maxima*** as a point on the hyperbola; many other points lay on the same hyperbola.

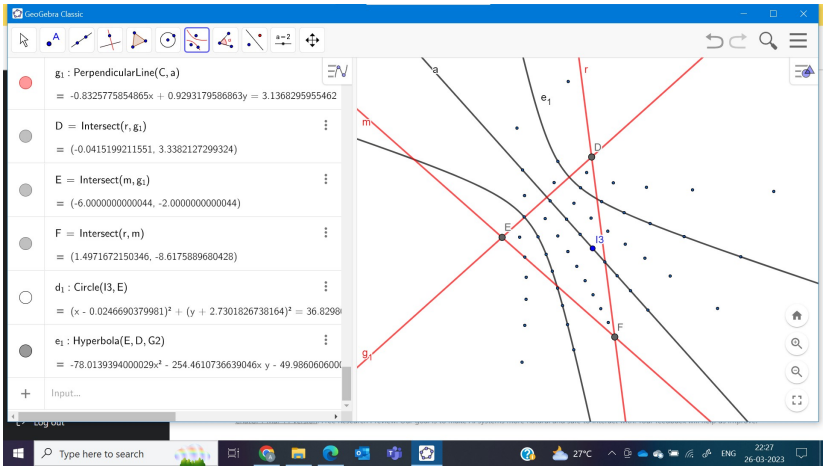

Figure 4: Verification

3 Numerical value of coordinates of sources

$$D \equiv (-0.0415, 3.3382)$$

$$E \equiv (-6, -2)$$

$$F \equiv (1.4971, -8.6175)$$

4 Appendix: A nice observation

As we already said, the three conjugate axes were the locus of points whose distance from two of the sources is equal. So the point where they intersect is equidistant from all the sources. That point is marked $I3$ in figure 5. In other words, all three sources must lie on a circle with $I3$ as the centre. I have verified this fact in figure 5 (black lines are the conjugate axes, and red lines are the transverse axes).

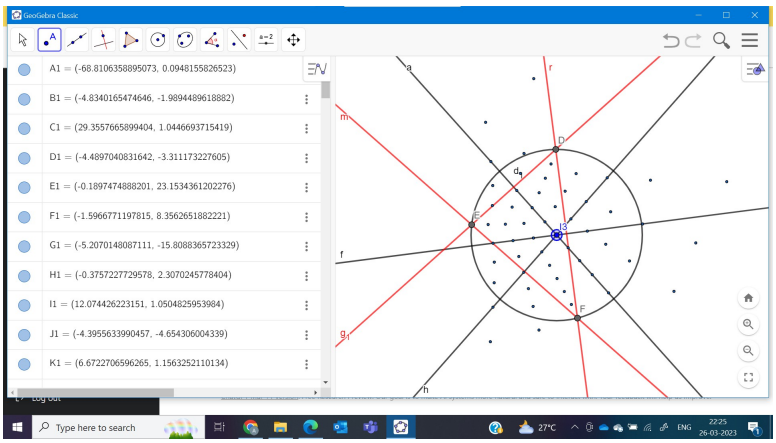

Figure 5: The circle as mentioned in appendix